Finite Automata

Part Two

Time-Out For Announcements!

A Note On Honor Code

- TAs have been noting down proofs written using AI and instances of copied work.
- I will go through the cases at the end of the quarter and report to the Office of Community Standards as needed. Any confirmed case would result in a zero in the class.
- Email me if you have violated the honor code, and we can instead give zeros on the affected PSETs.

Back to CS103!

Just how powerful are NFAs?

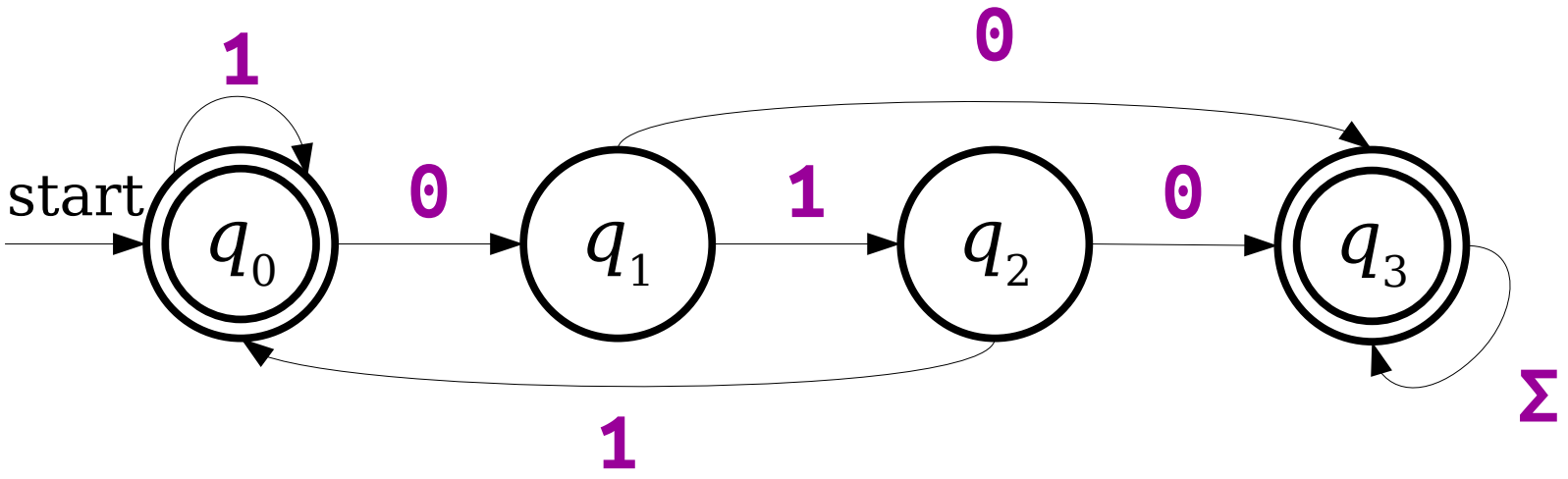
NFAs and DFAs

- Any language that can be accepted by a DFA can be accepted by an NFA.
- Why?
  - Every DFA essentially already *is* an NFA!
- ***Question:*** Can any language accepted by an NFA also be accepted by a DFA?
- Surprisingly, the answer is ***yes***!

Thought Experiment:

How would you simulate a finite automata in software?

Tabular DFAs



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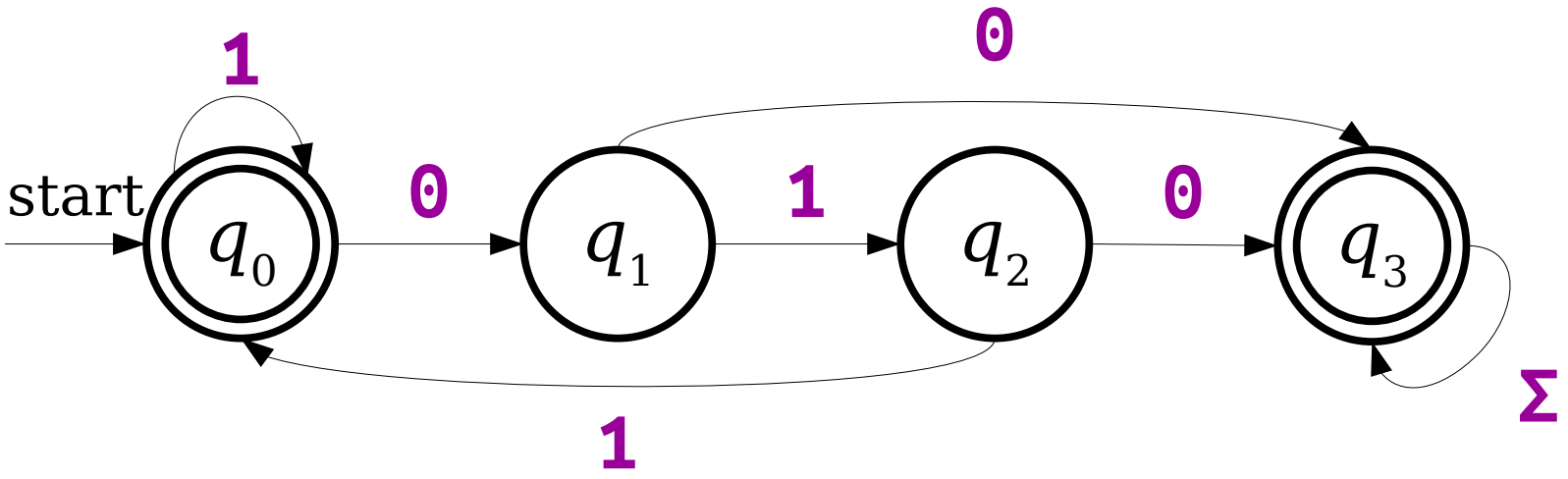
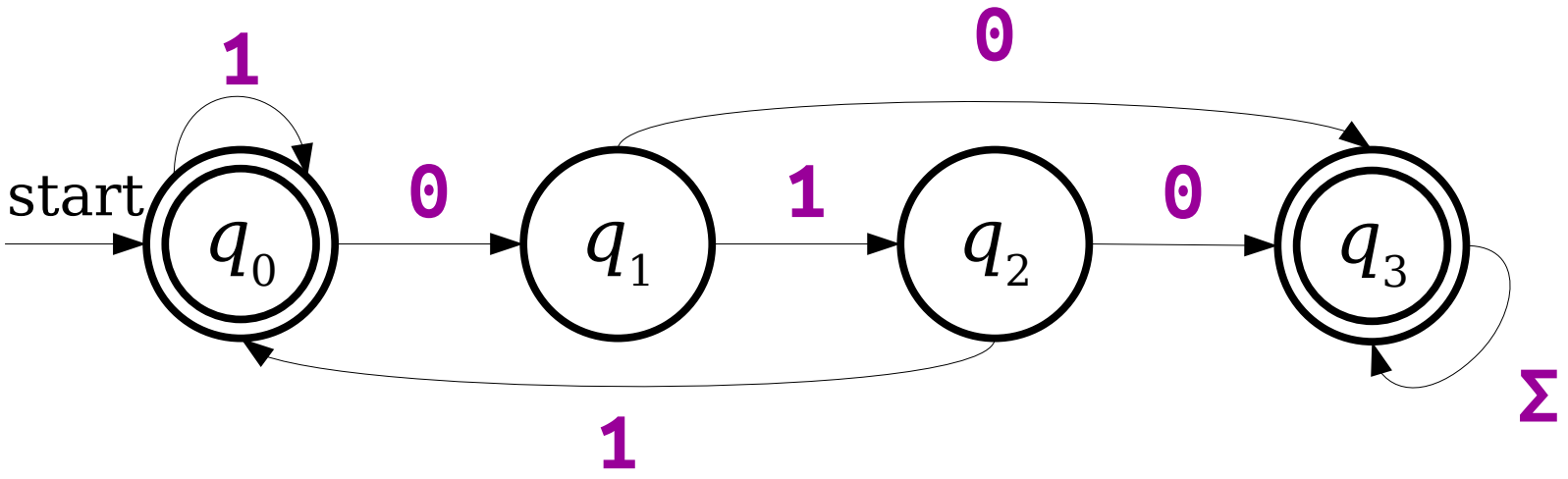
q_0		
q_1		
q_2		
q_3		

Tabular DFAs



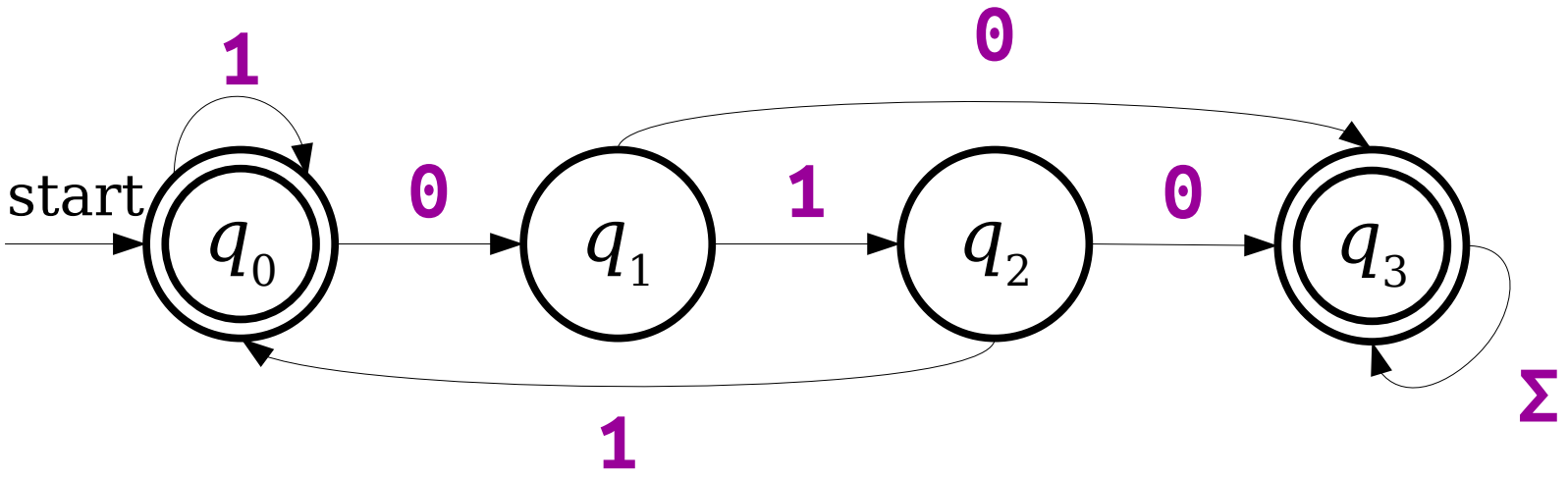
	0	1
q_0	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
q_3	q_3	q_3

Tabular DFAs



	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

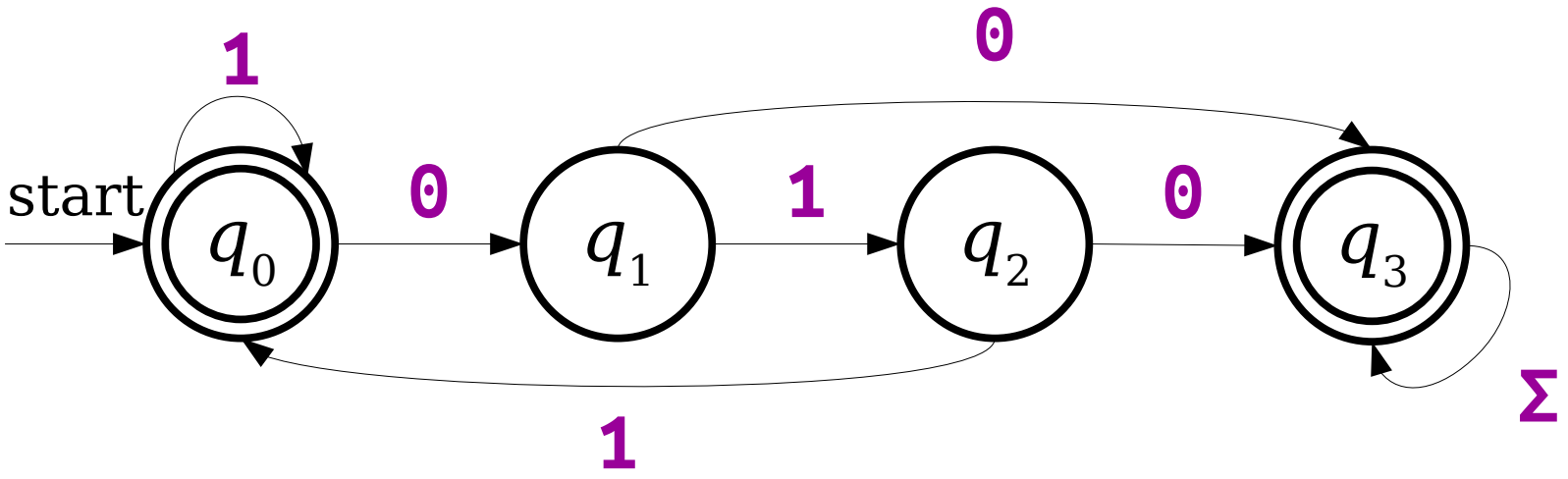
Tabular DFAs



	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

These stars indicate accepting states.

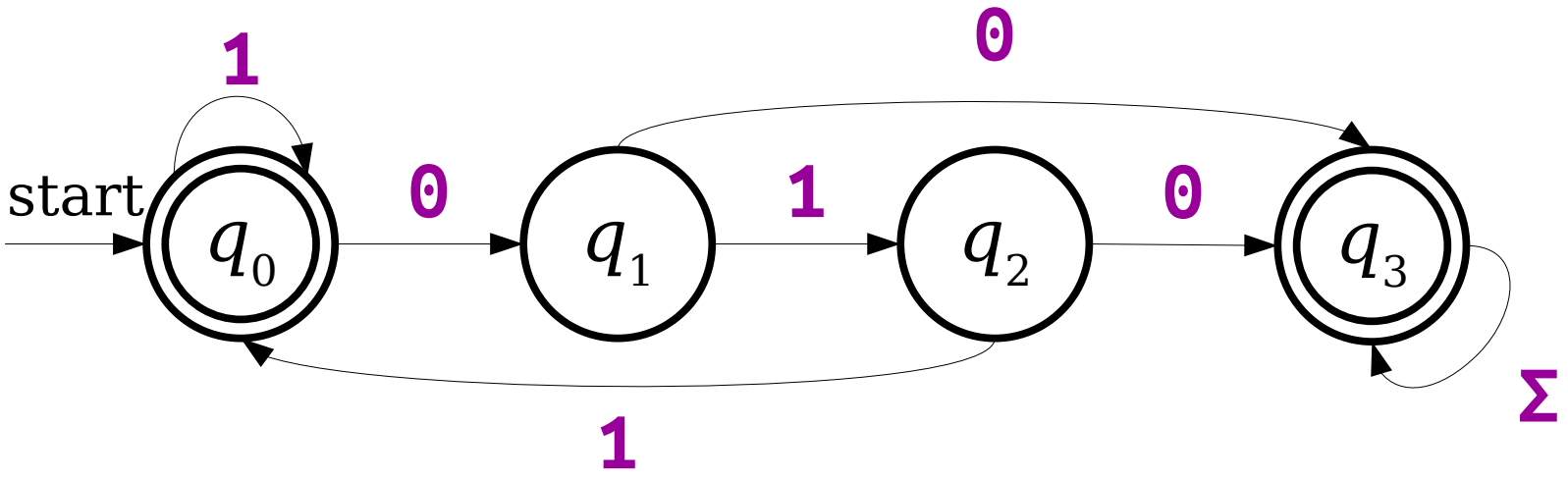
Tabular DFAs



	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

Since this is the first row, it's the start state.

Tabular DFAs



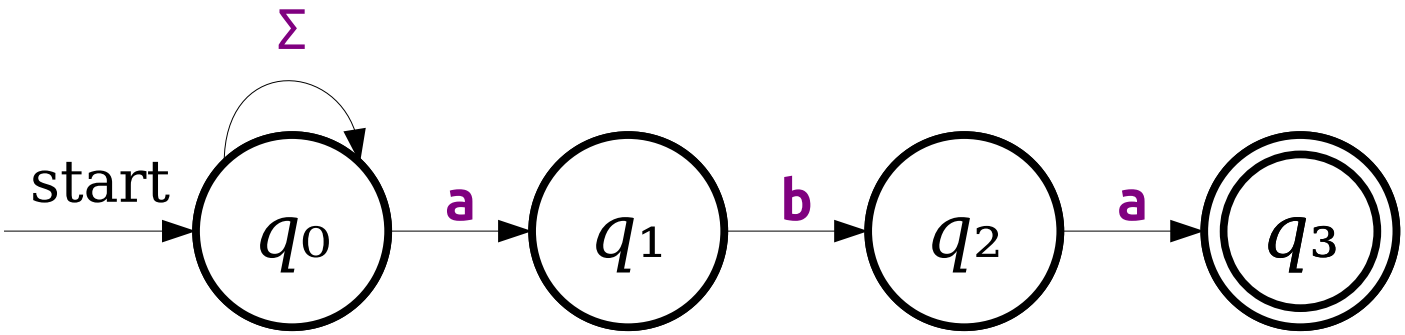
	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

Question to ponder:
Why isn't there a column here for Σ?

Code? In a Theory Class?

```
int kTransitionTable[kNumStates][kNumSymbols] = {
     {0, 0, 1, 3, 7, 1, …},
       …
};
bool kAcceptTable[kNumStates] = {
    false,
    true,
    true,
    …
};
bool SimulateDFA(string input) {
    int state = 0;
    for (char ch: input) {
        state = kTransitionTable[state][ch];
    }
    return kAcceptTable[state];
}
```

Can we do something similar for NFAs?


Σ
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q_0
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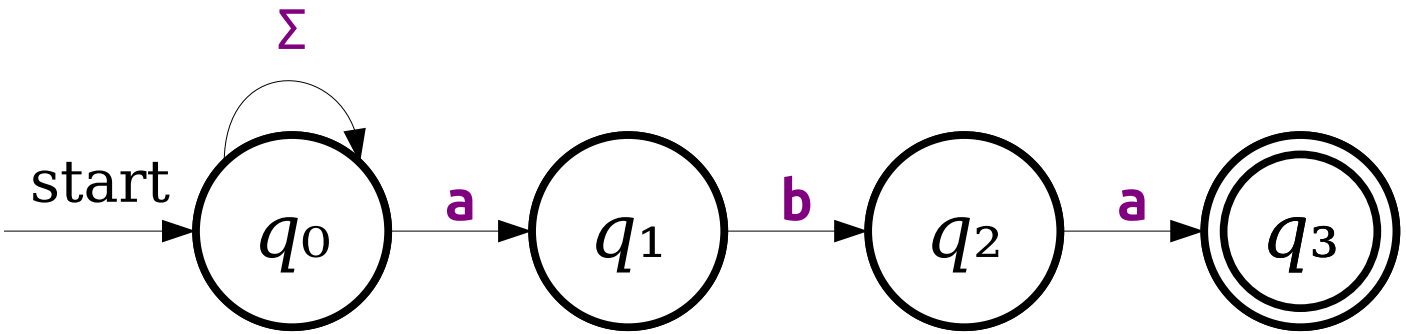
Σ
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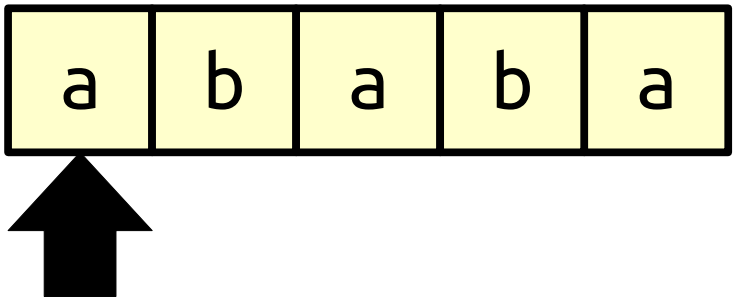
q_3


a
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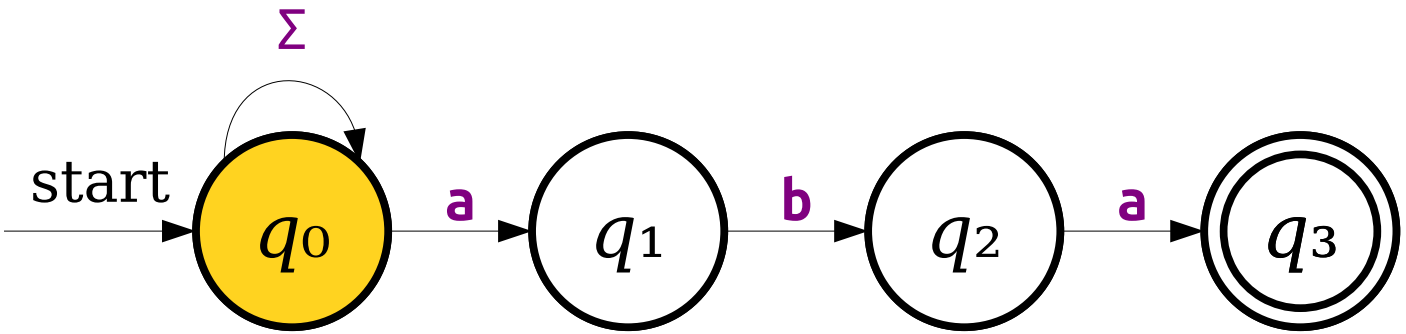
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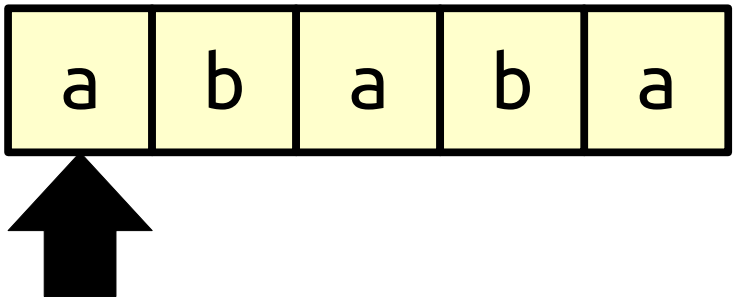
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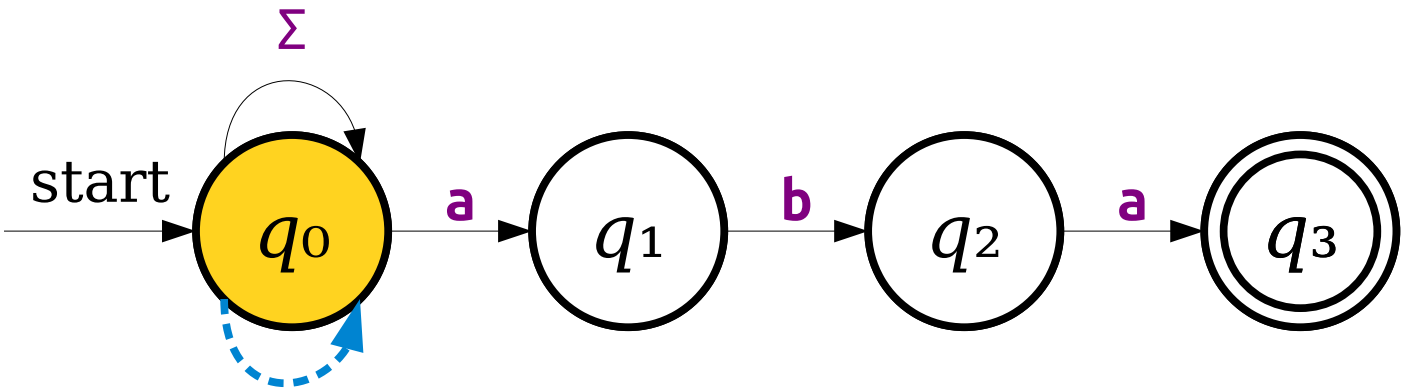
Σ
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q_0
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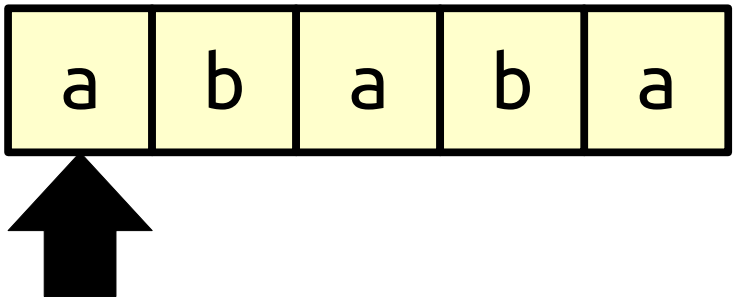
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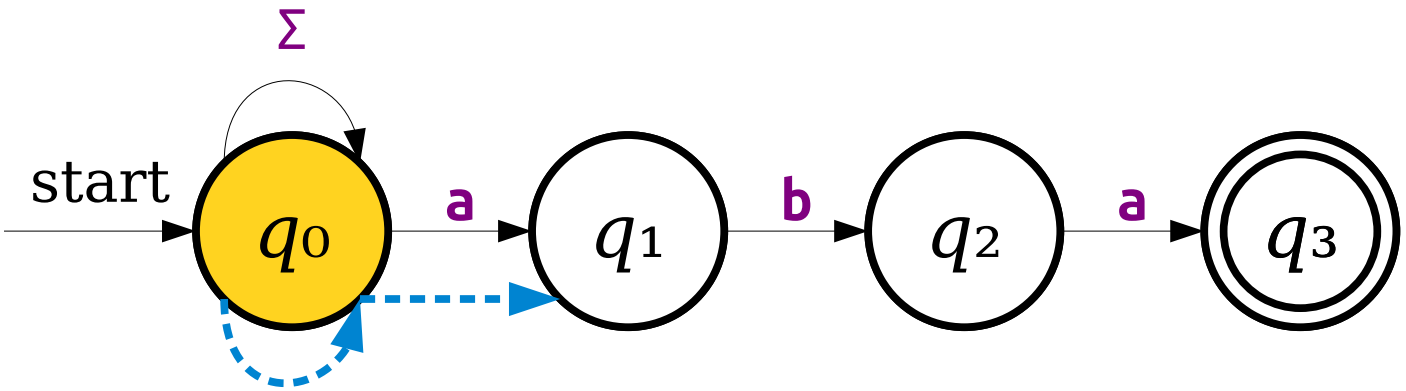
Σ
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q_0
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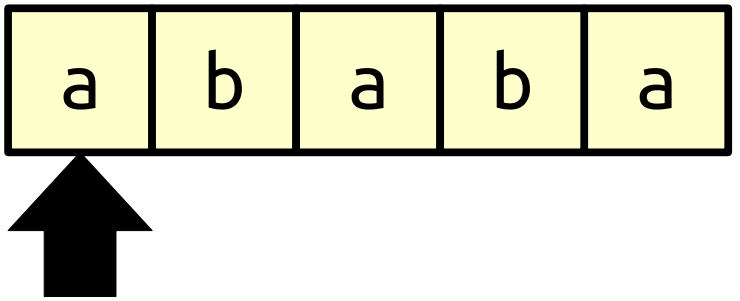
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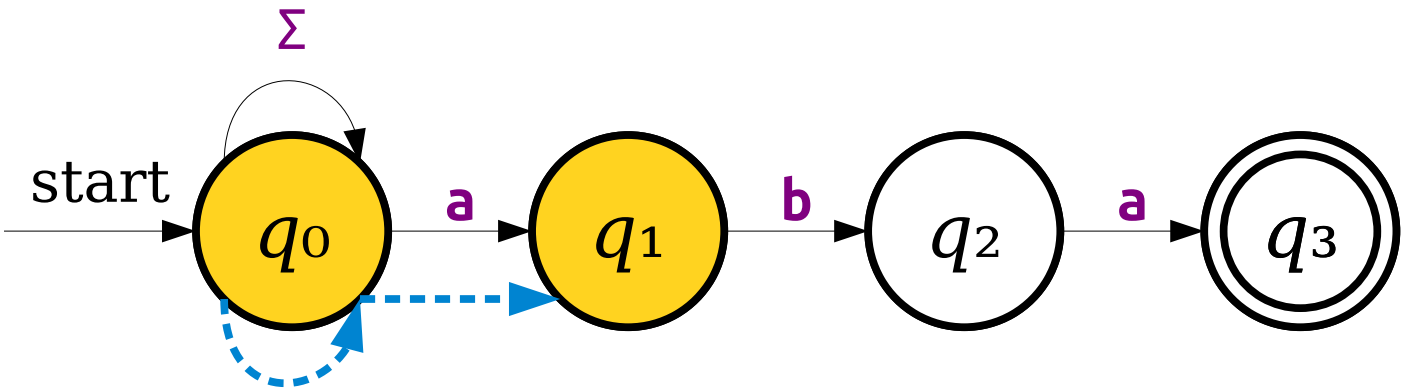
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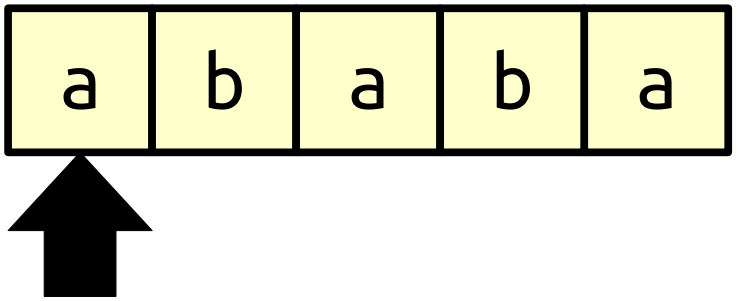
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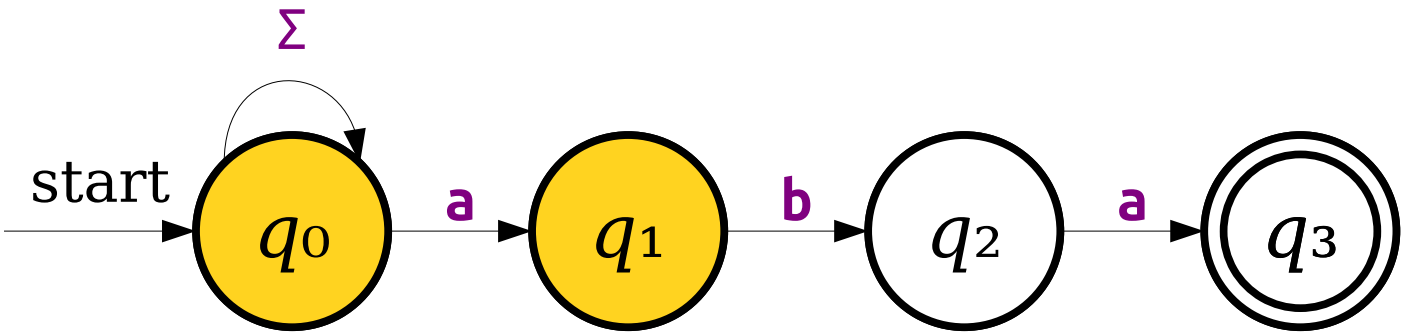
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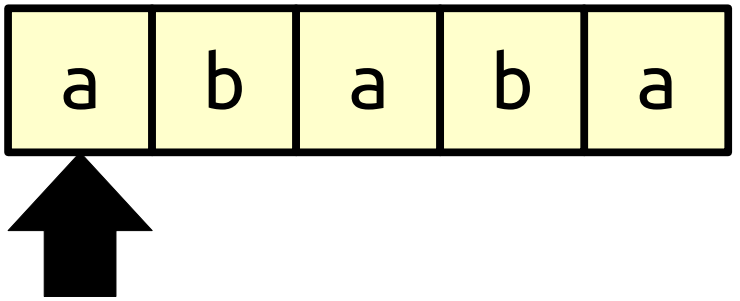
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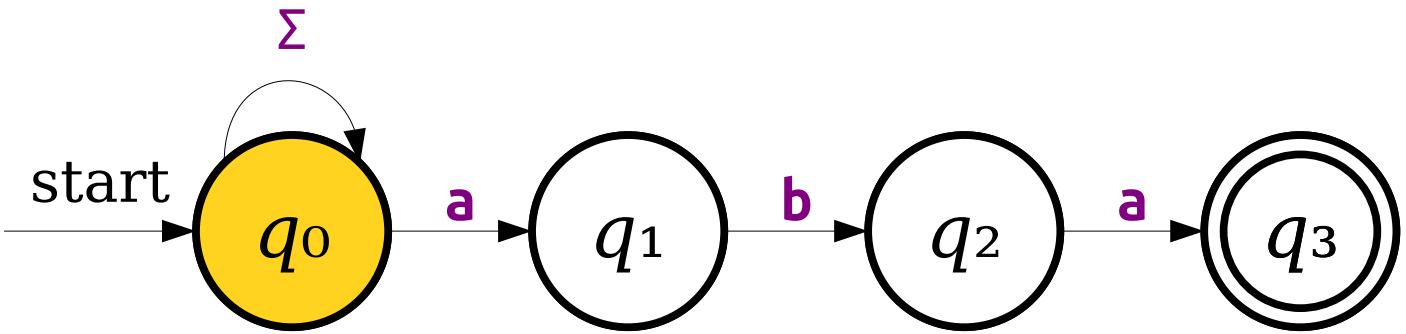
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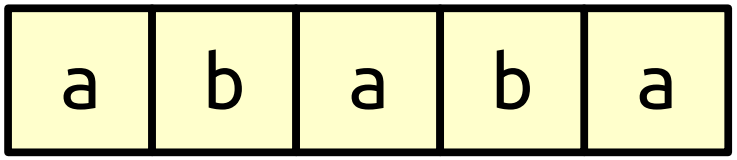
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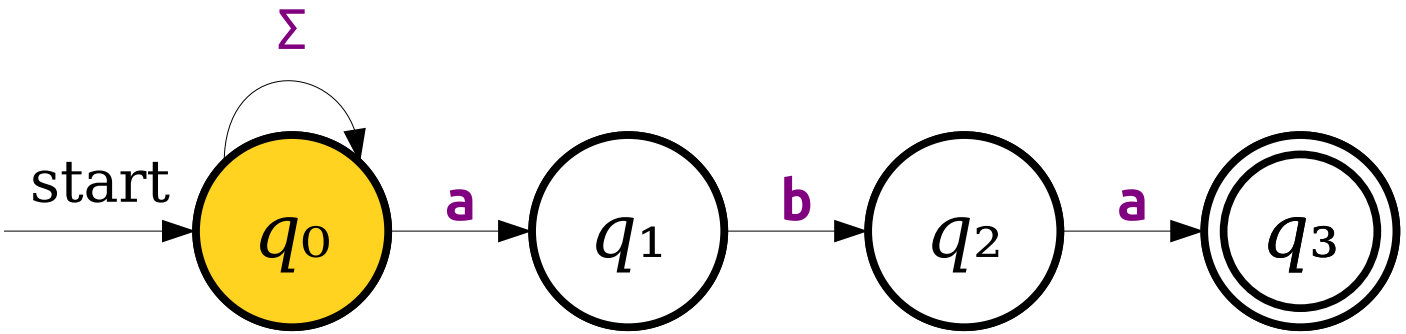
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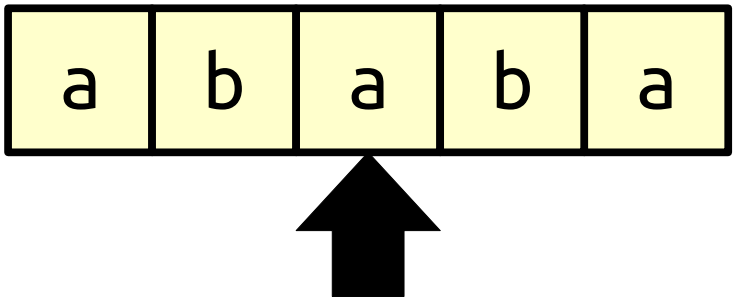
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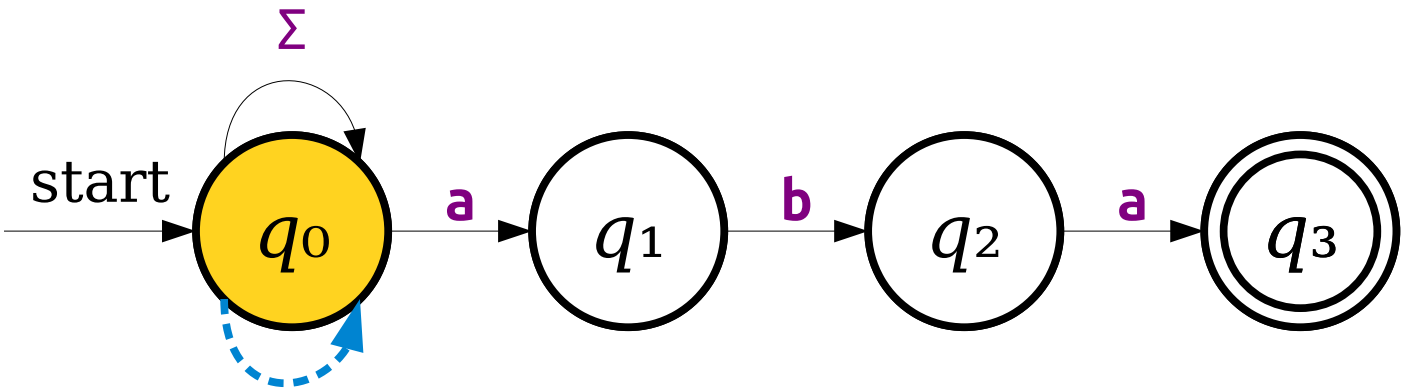
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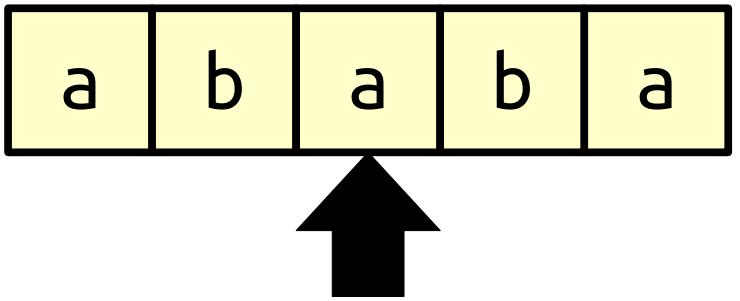
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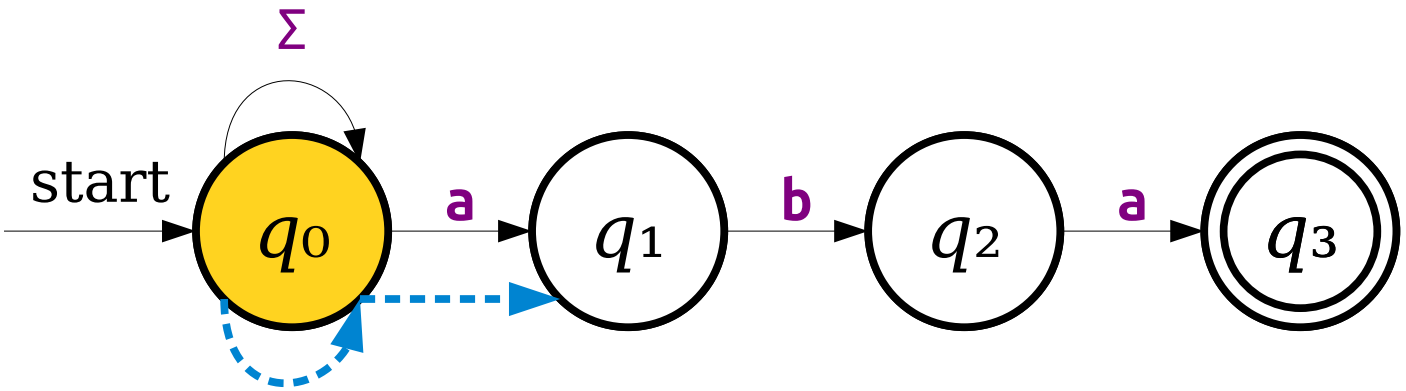
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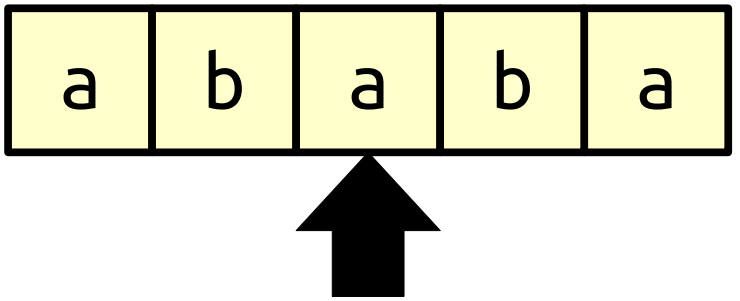
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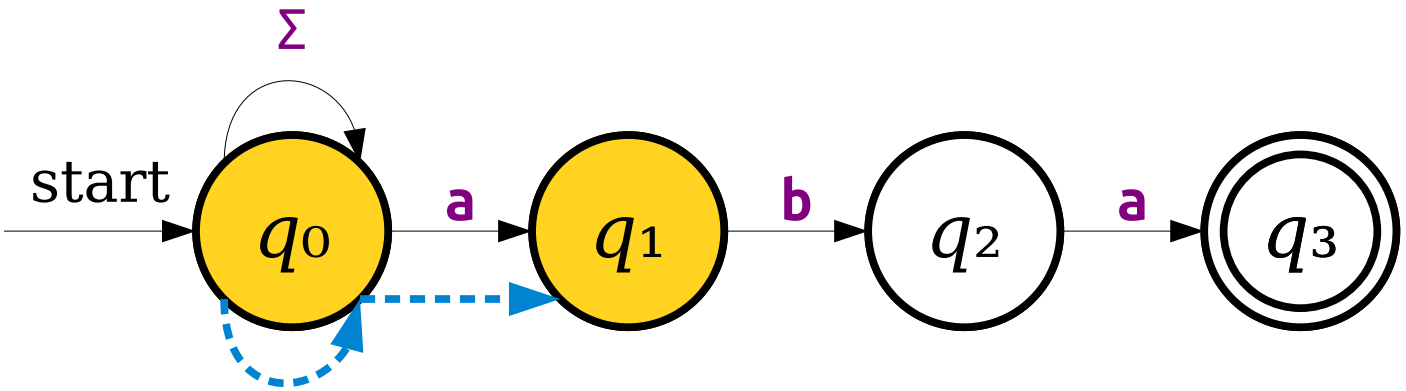
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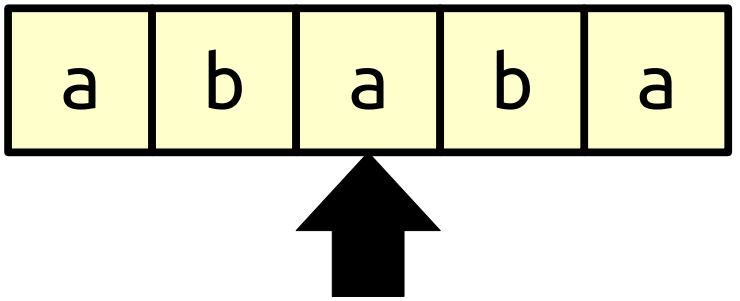
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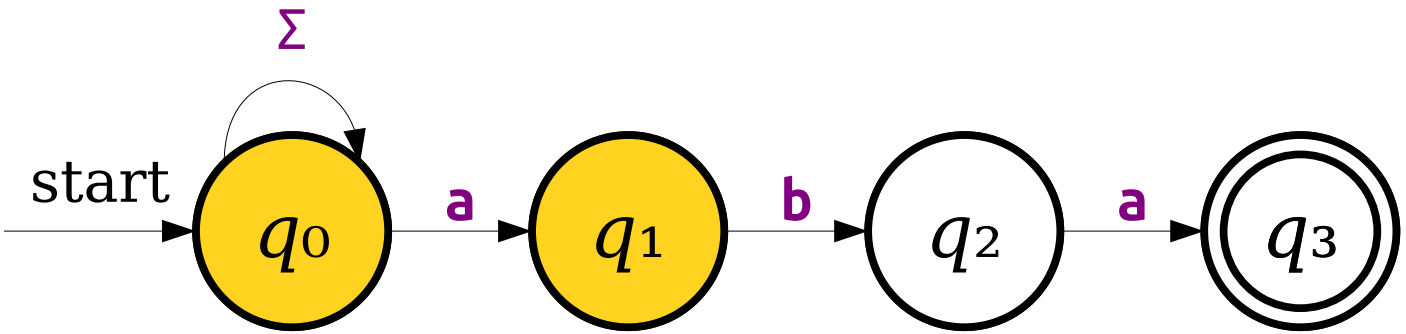
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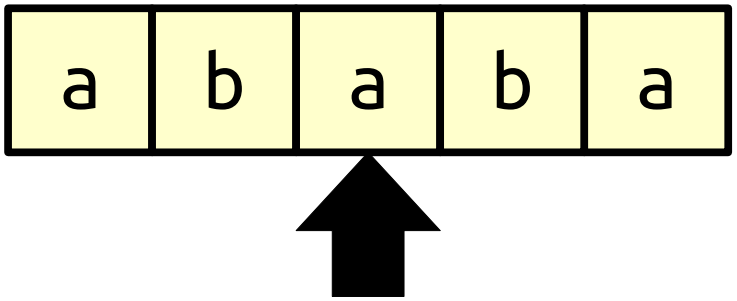
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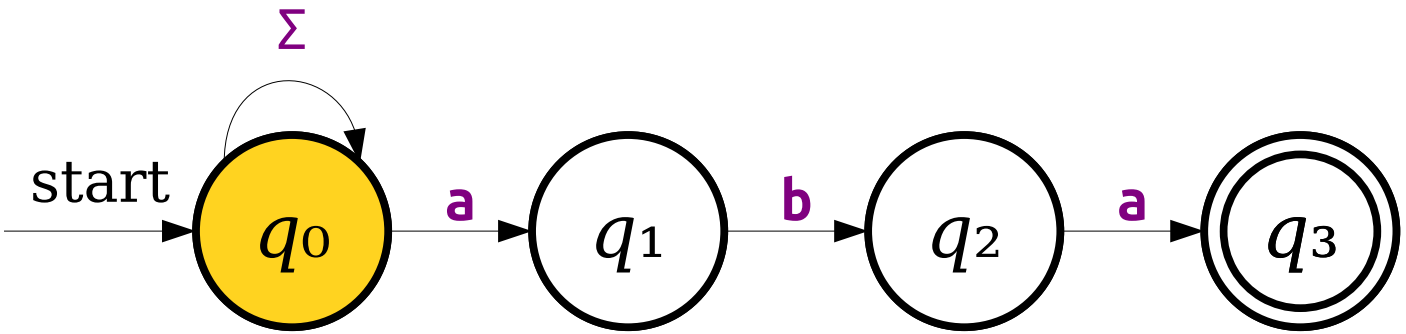
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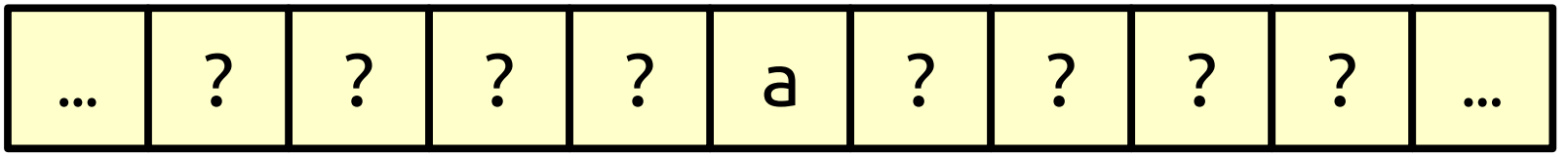
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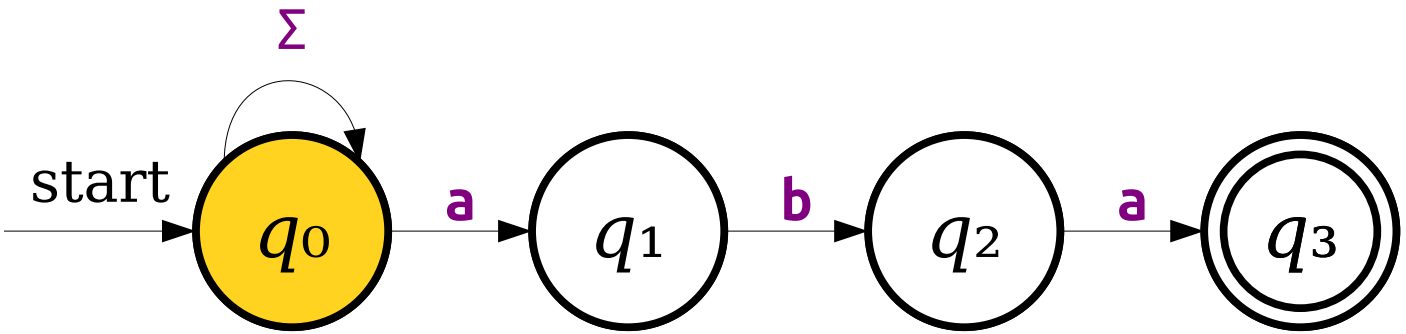
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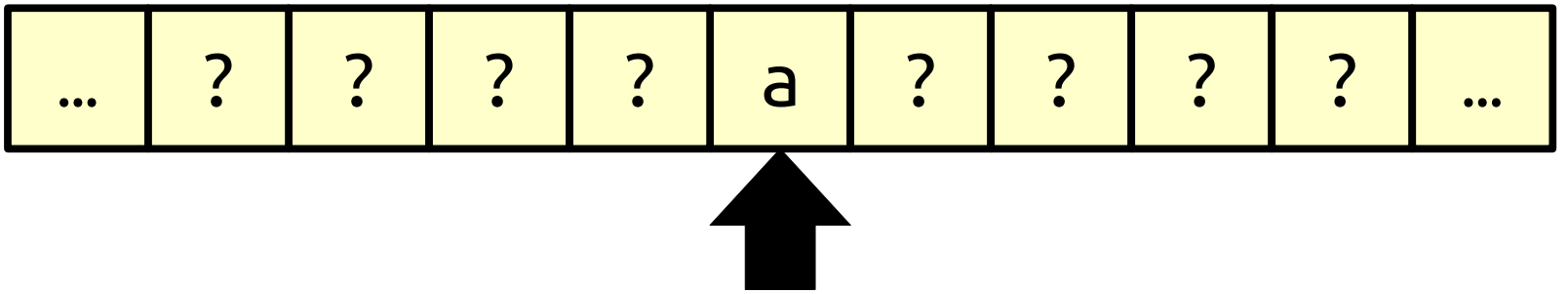
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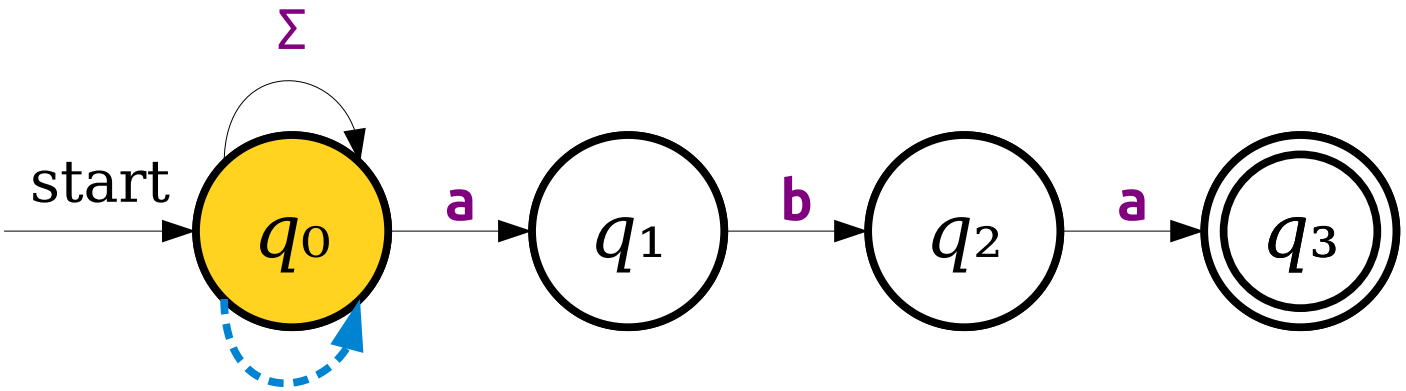
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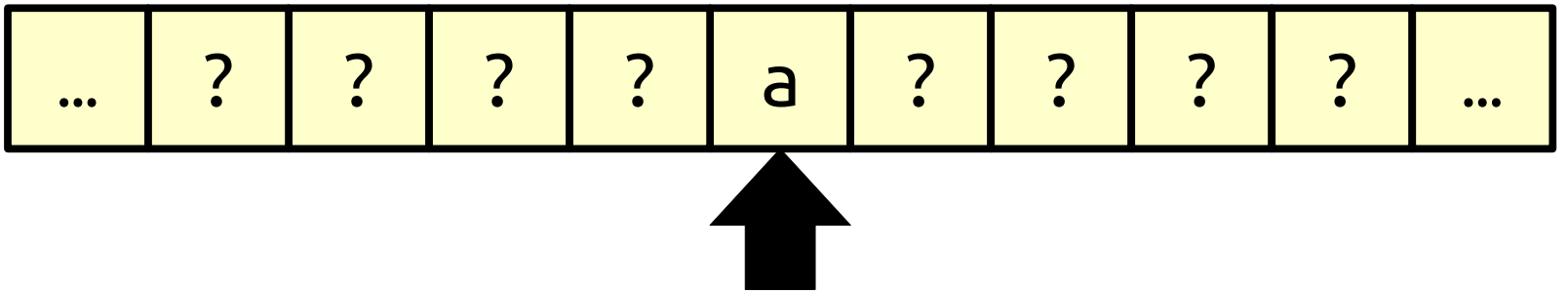
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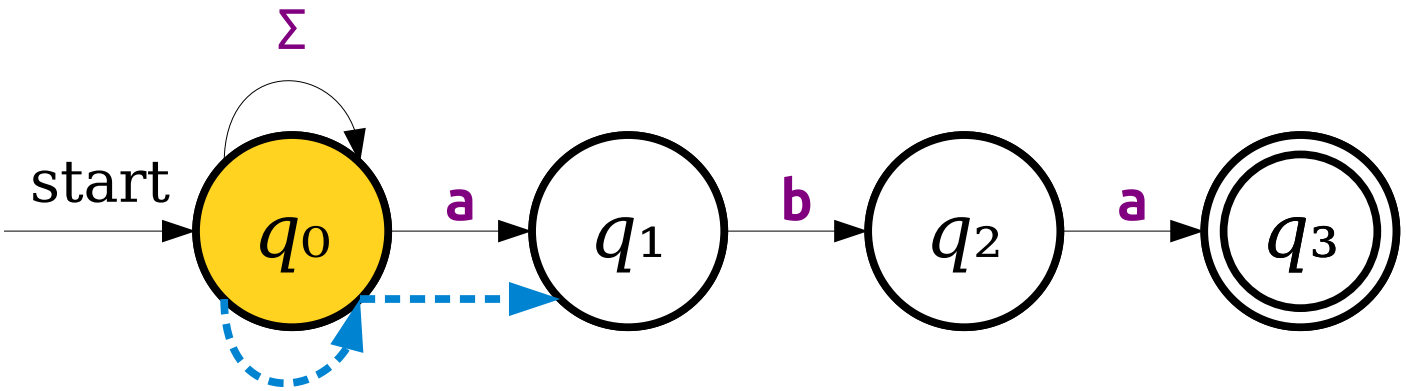
Σ
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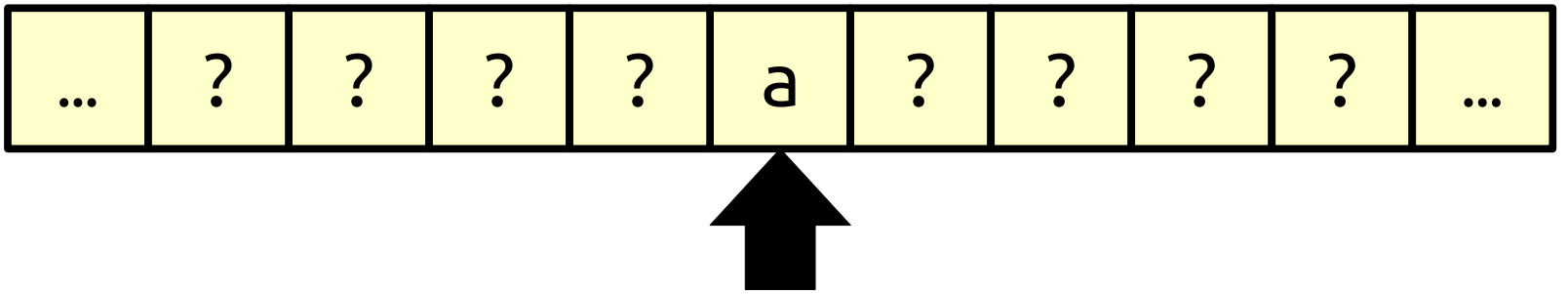
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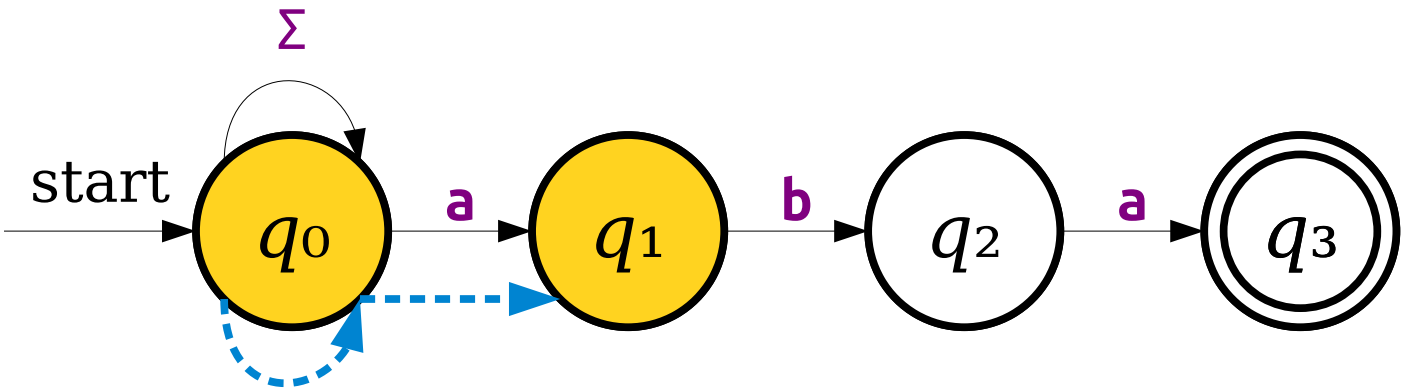
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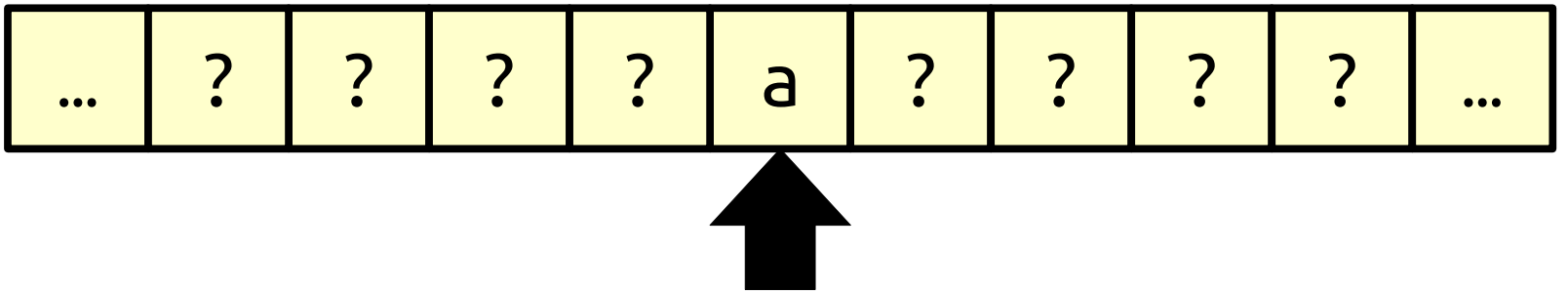
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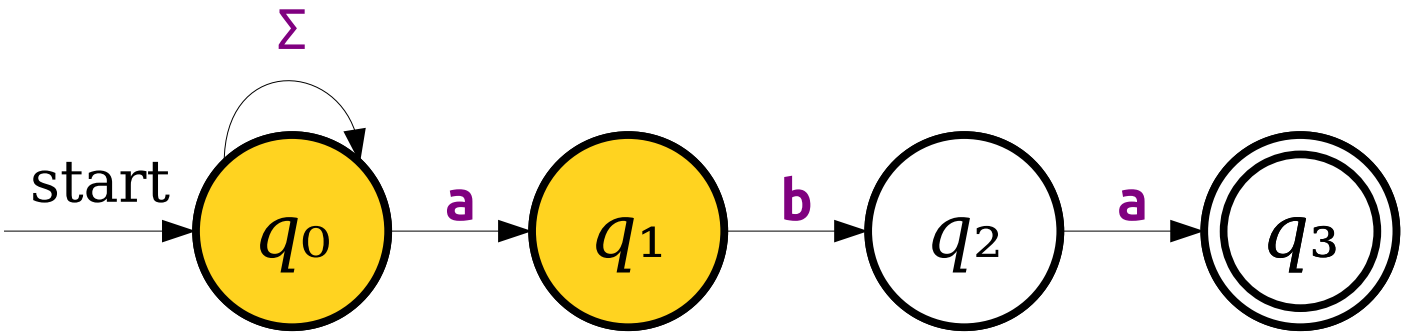
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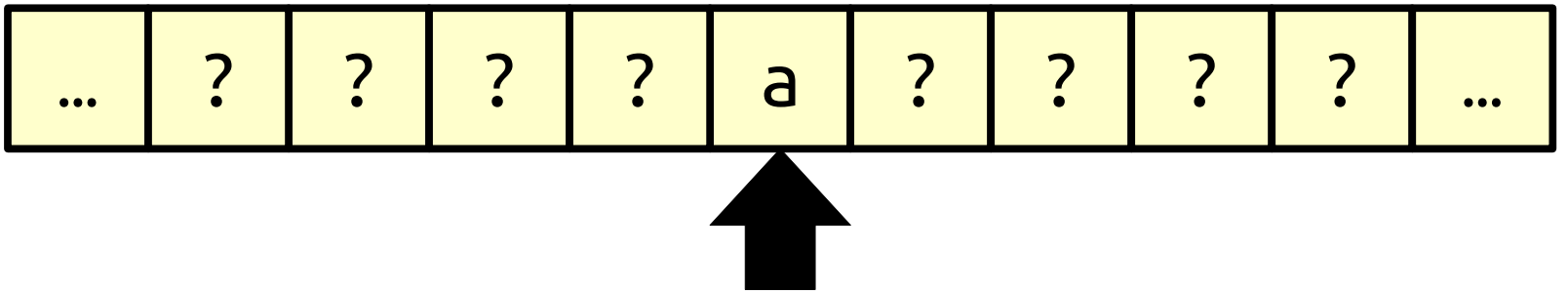
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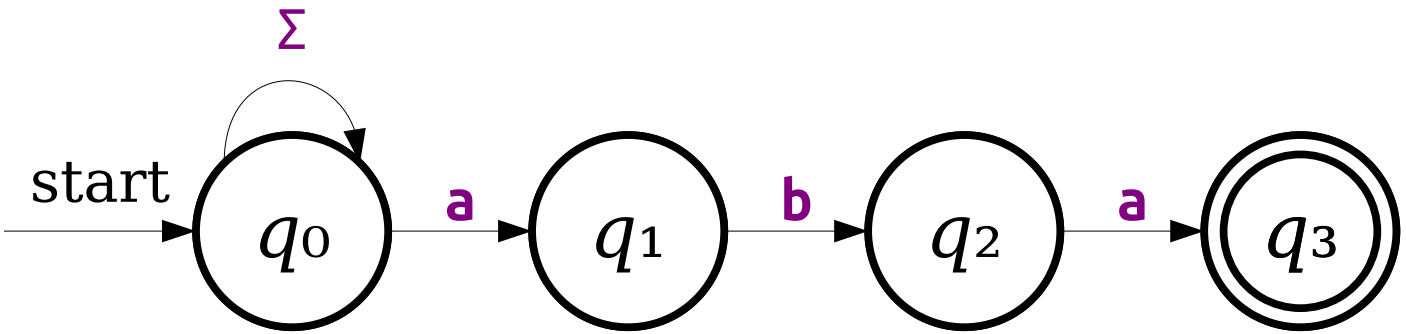
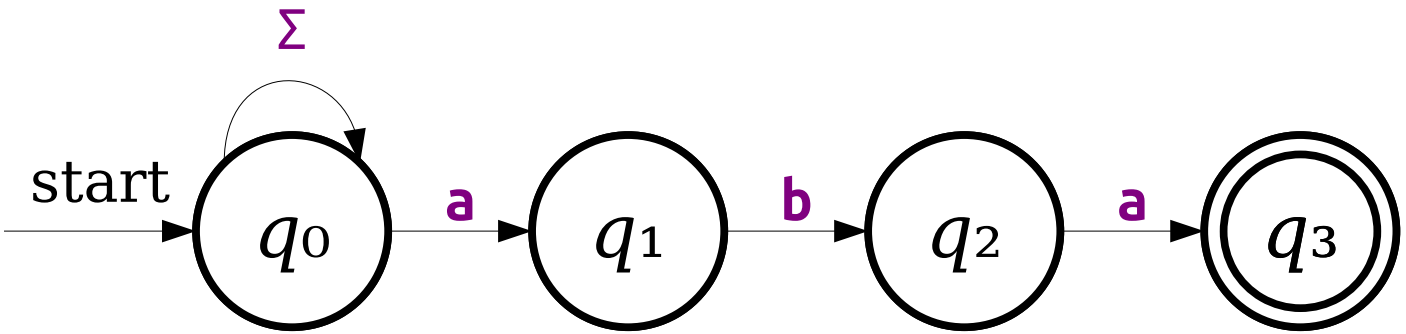
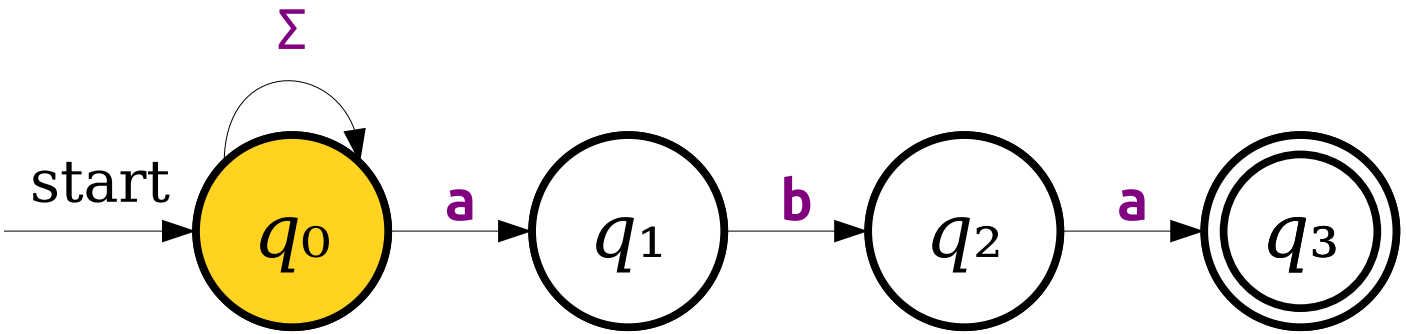


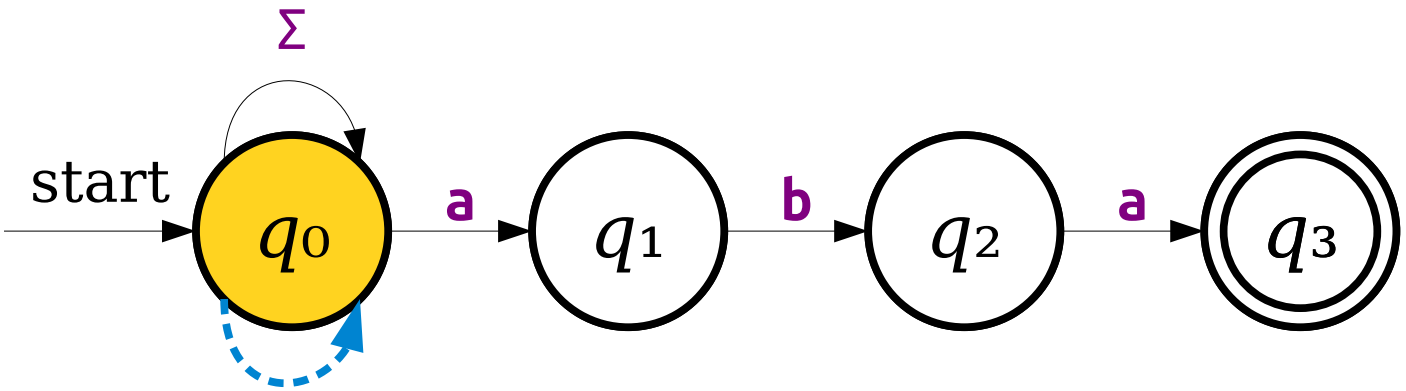
	a
$\{q_0\}$	$\{q_0, q_1\}$



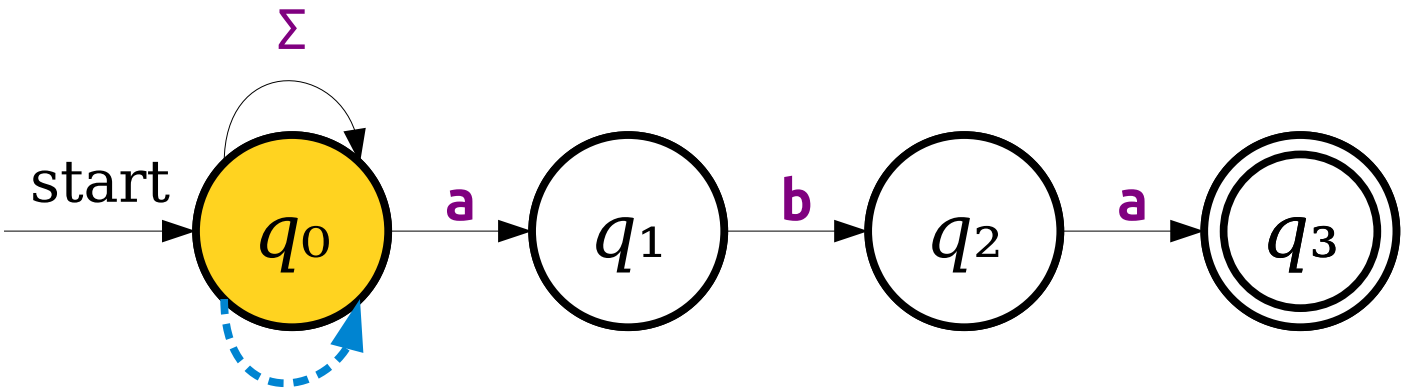
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	



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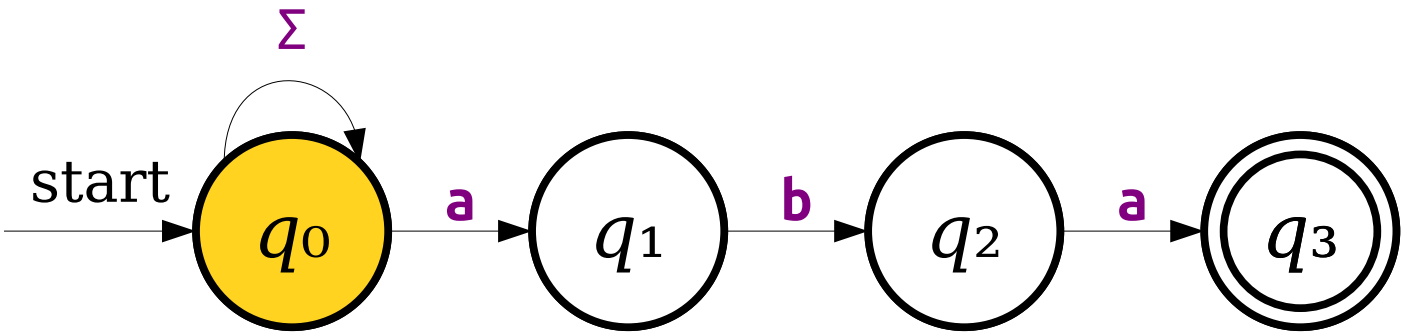

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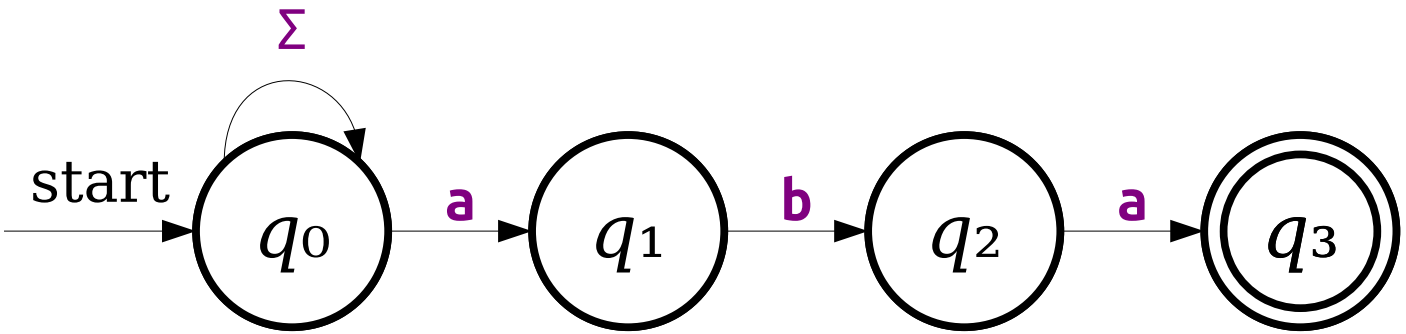

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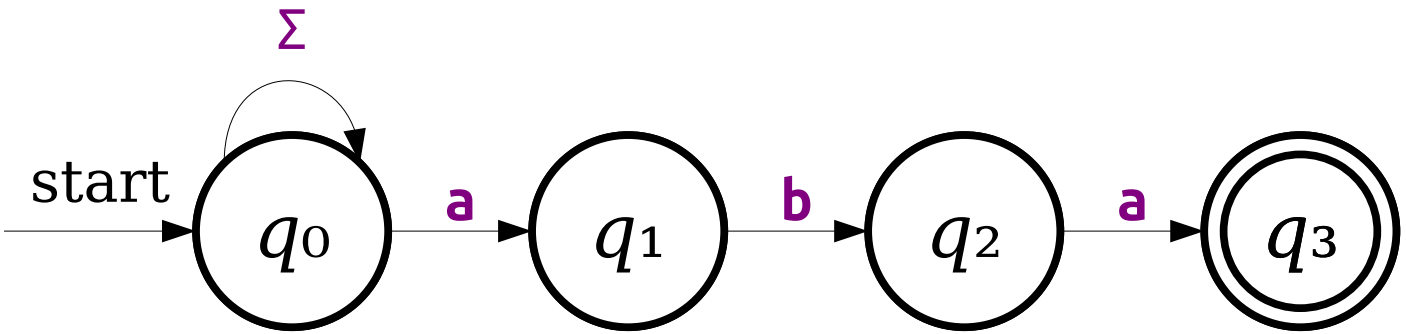

	a	b
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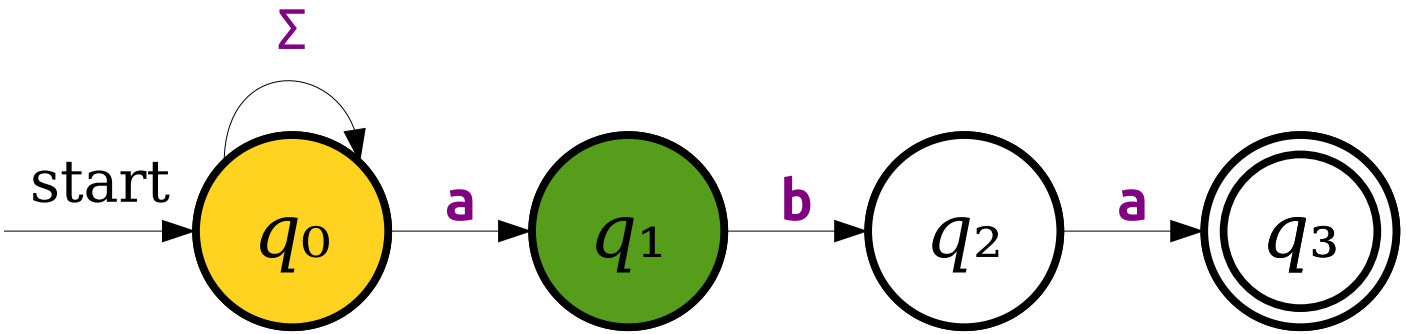

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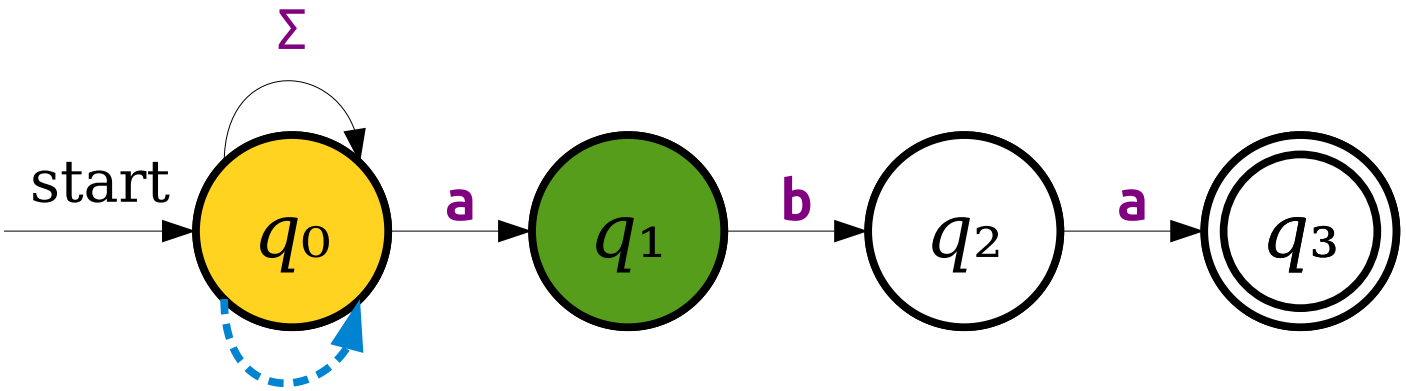

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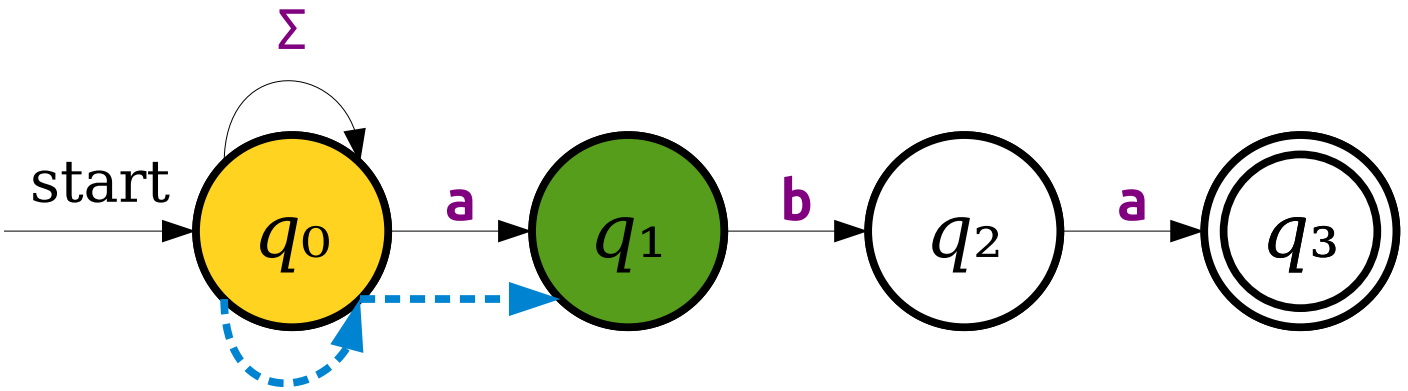

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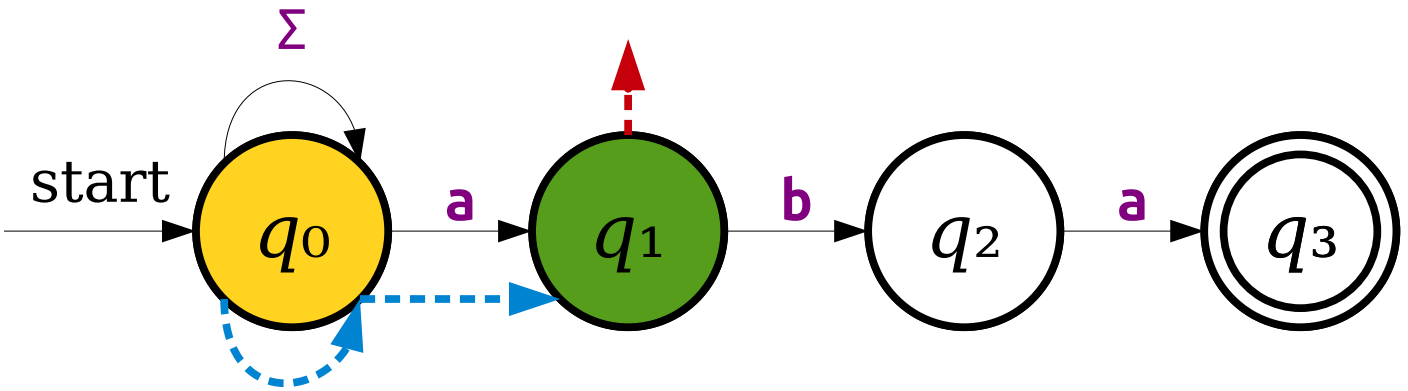

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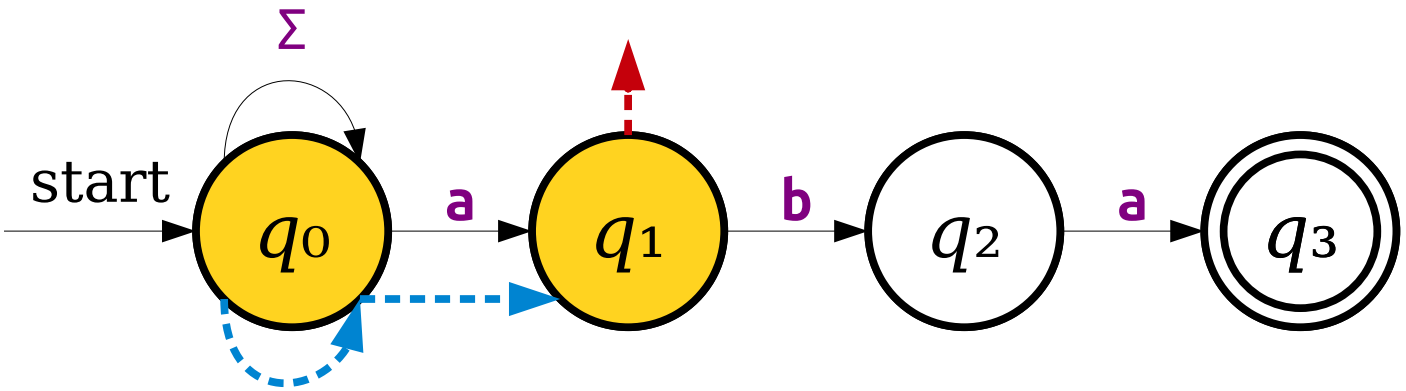

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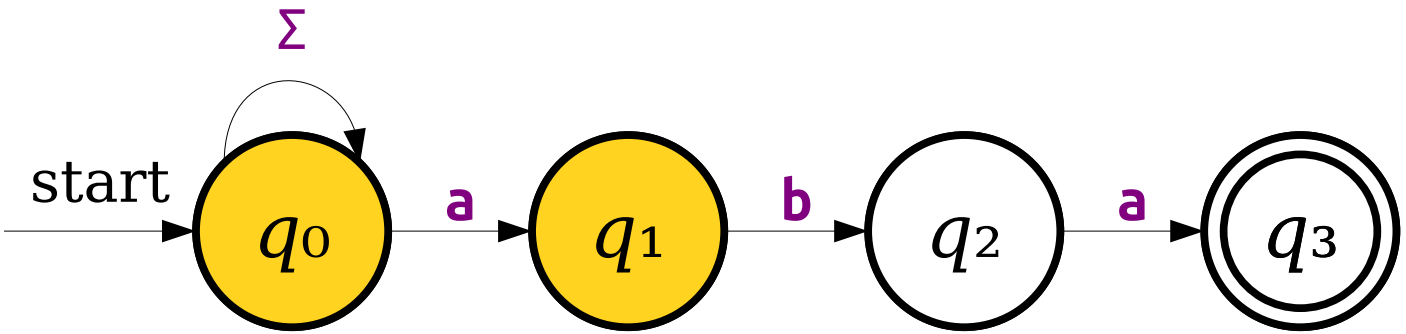

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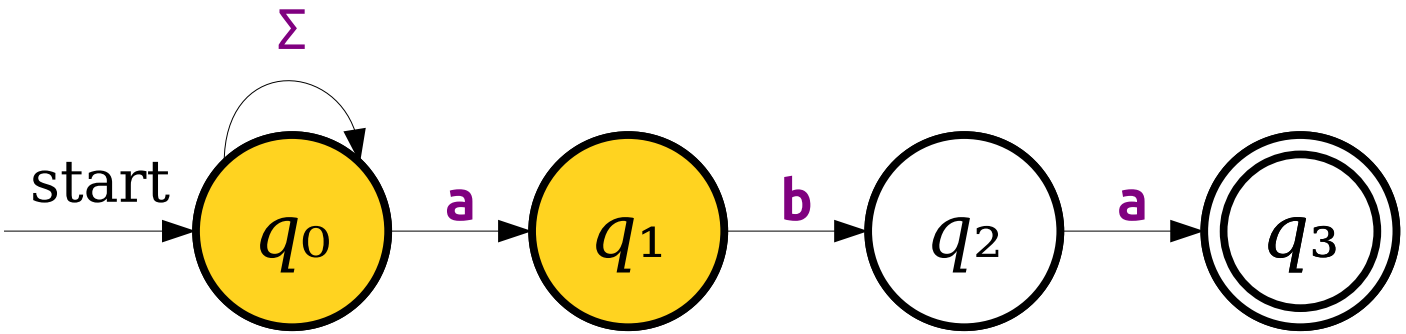

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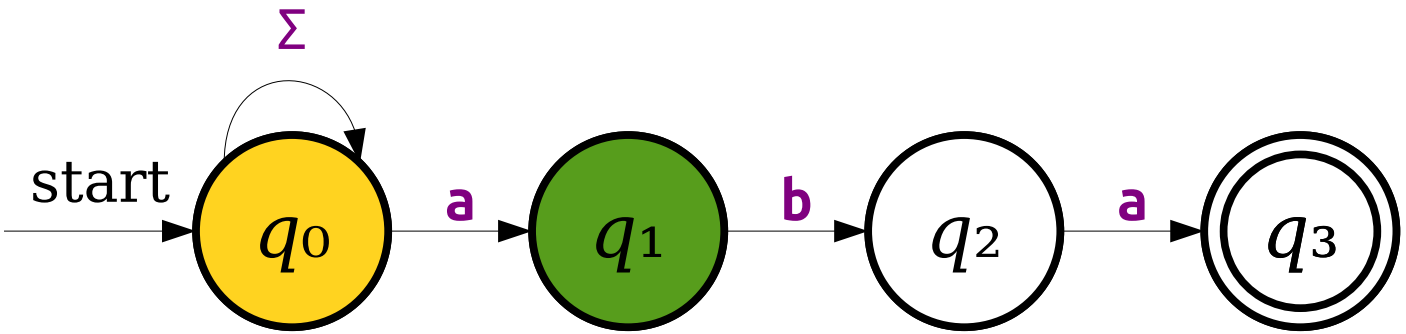

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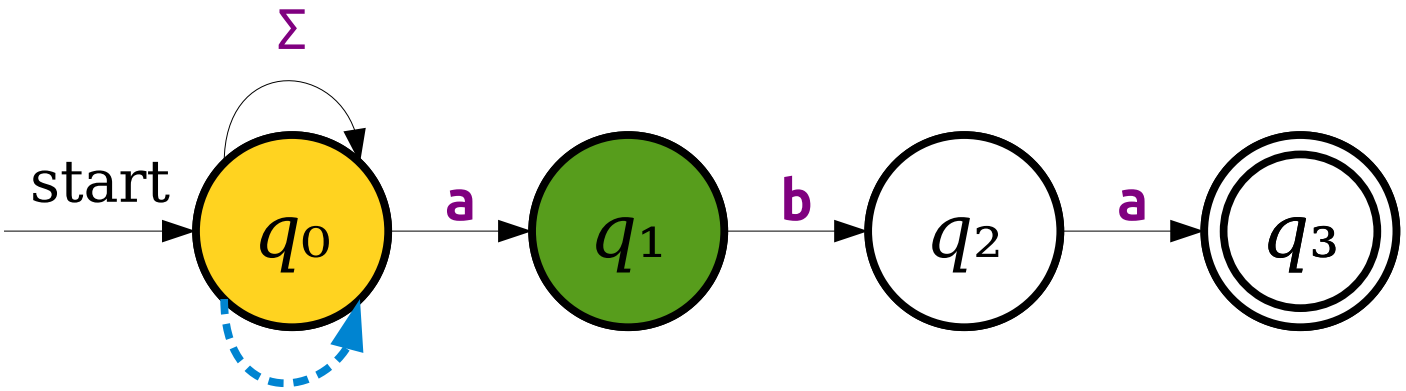

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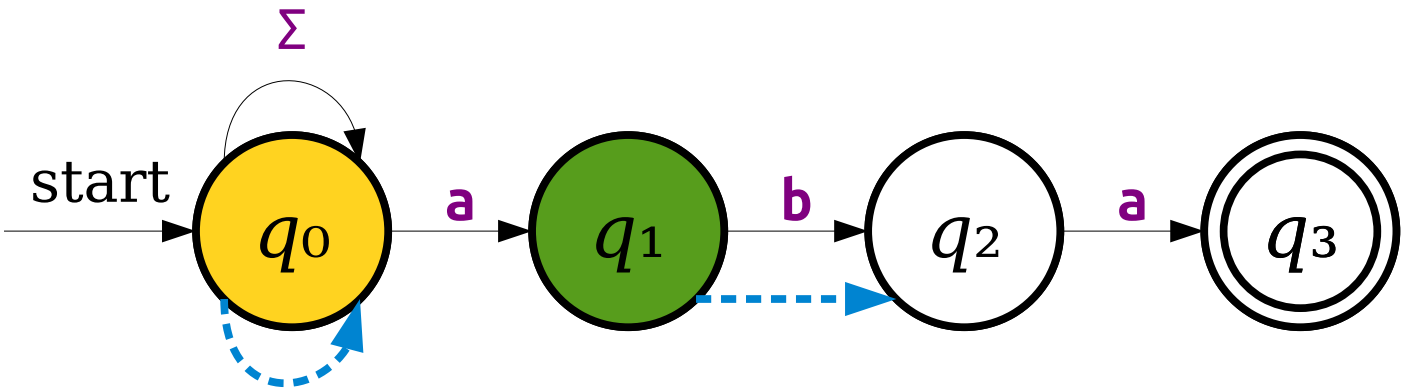

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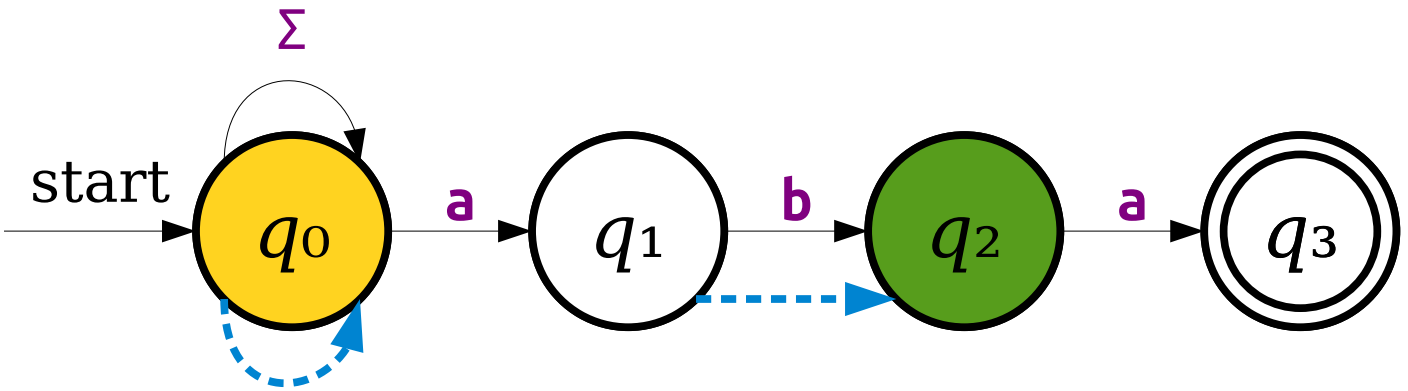

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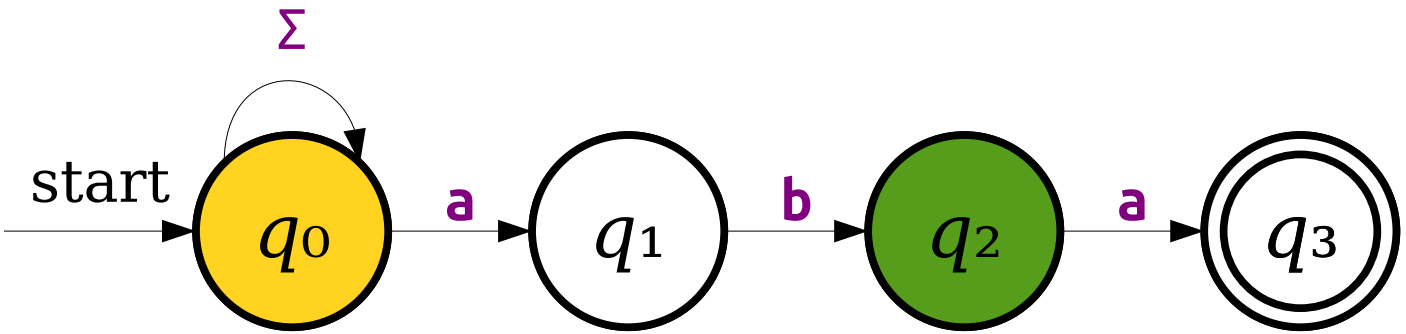

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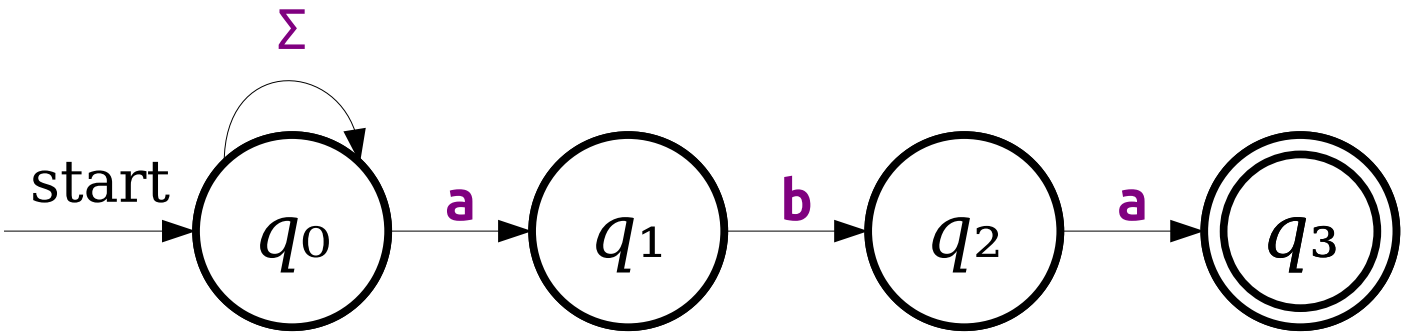

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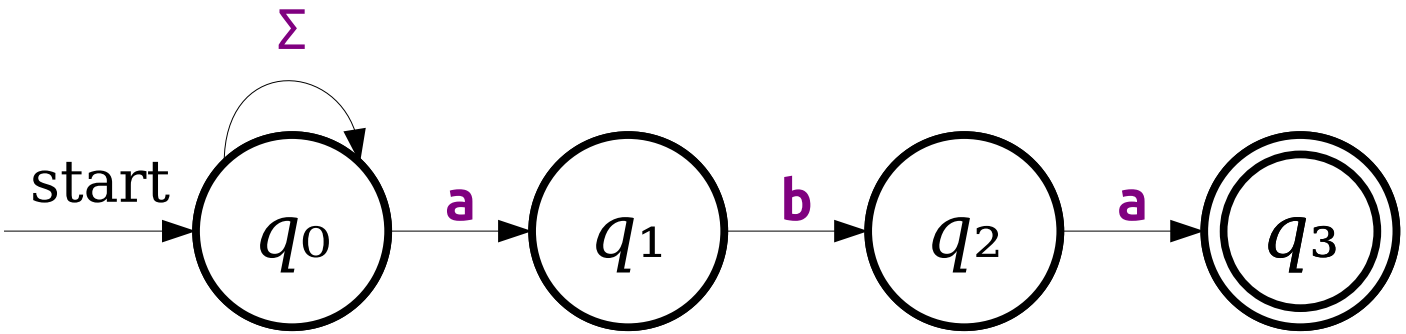

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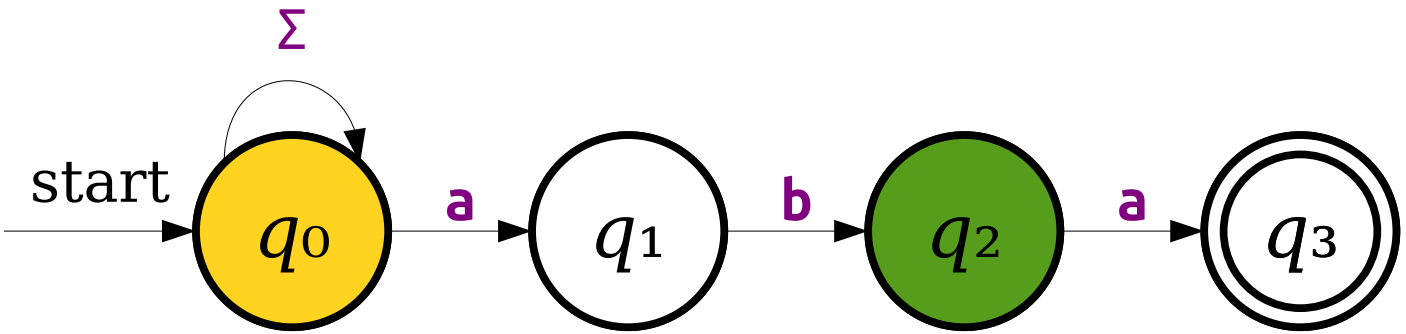

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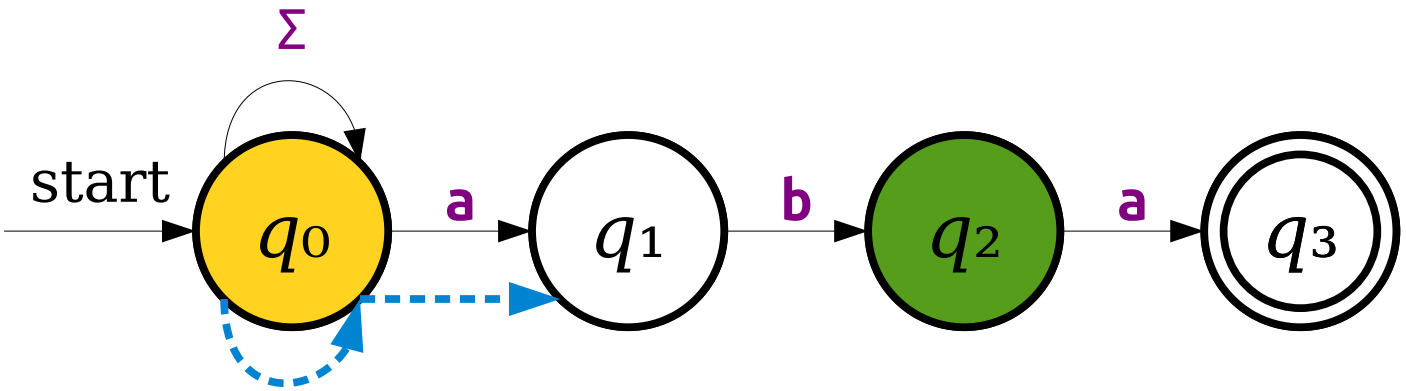

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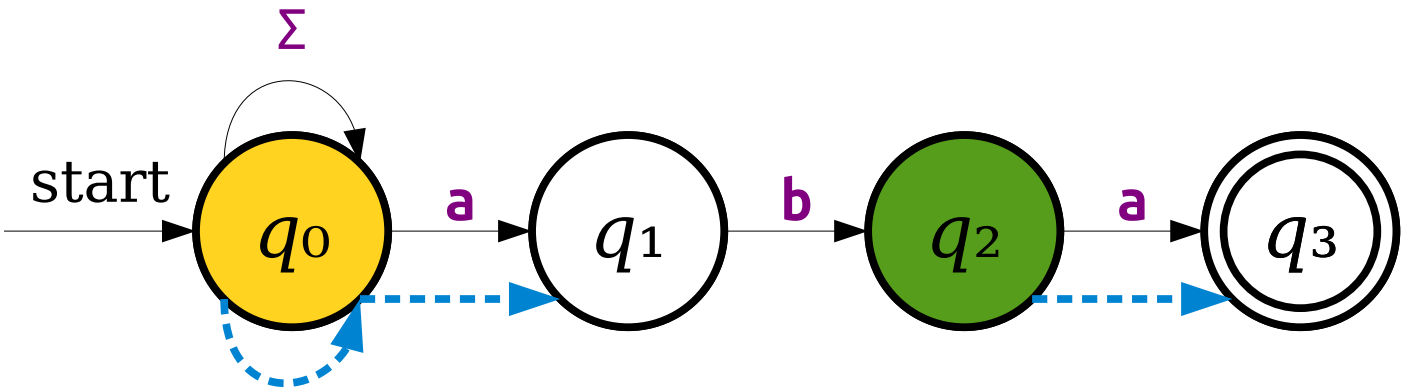

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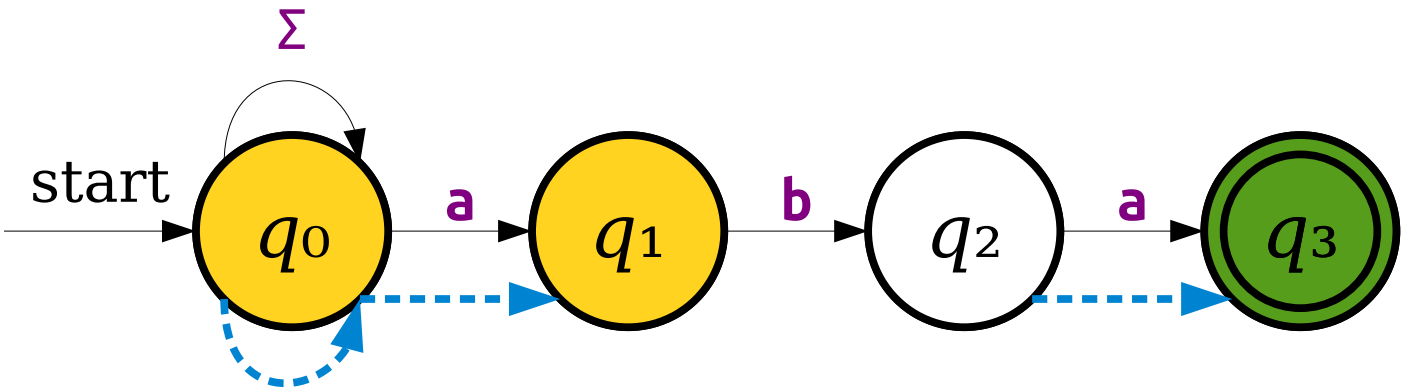

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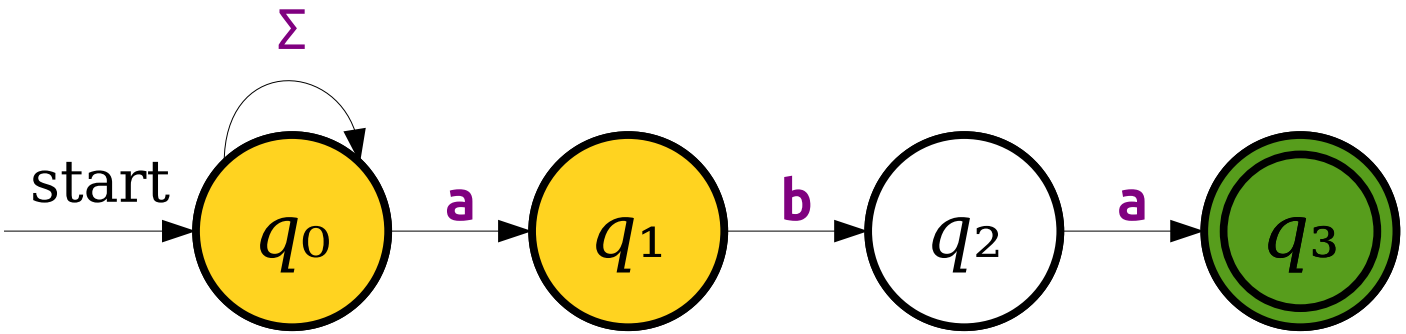

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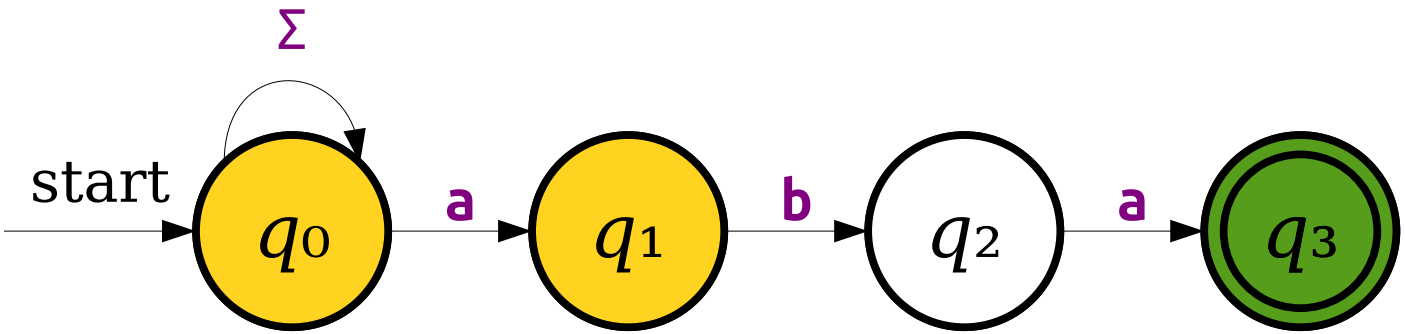

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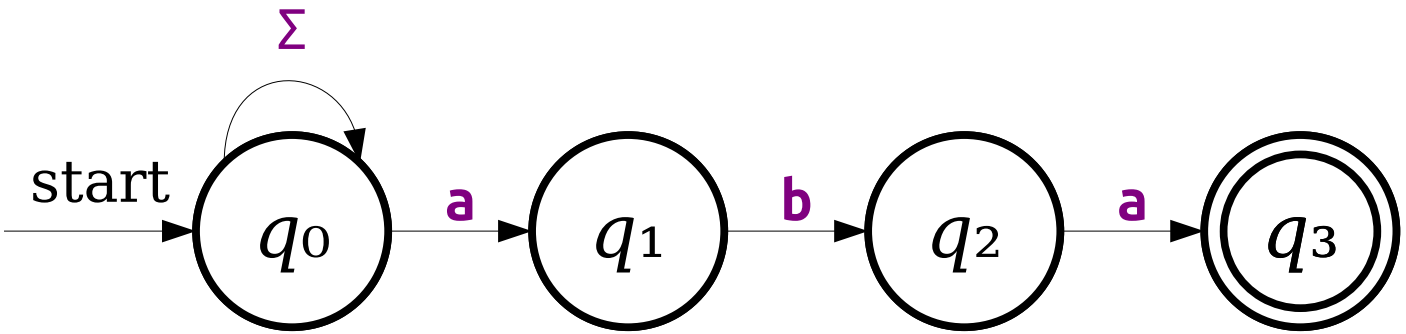

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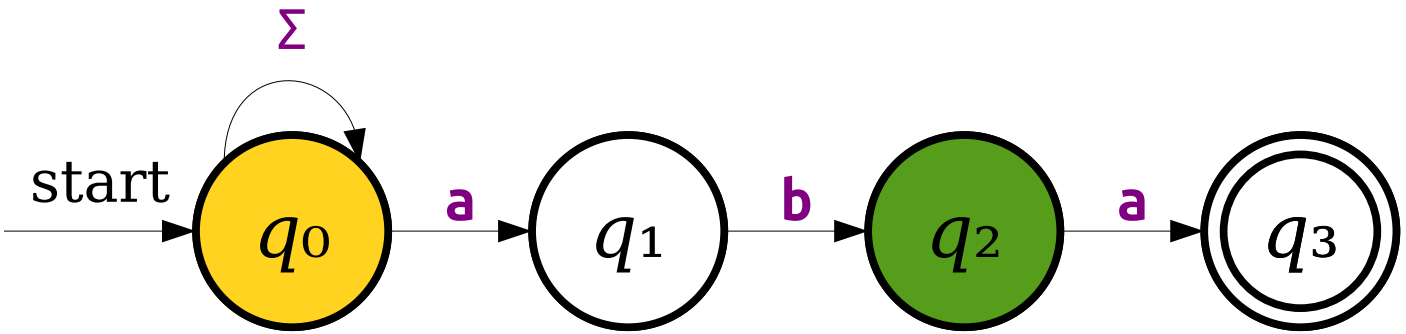

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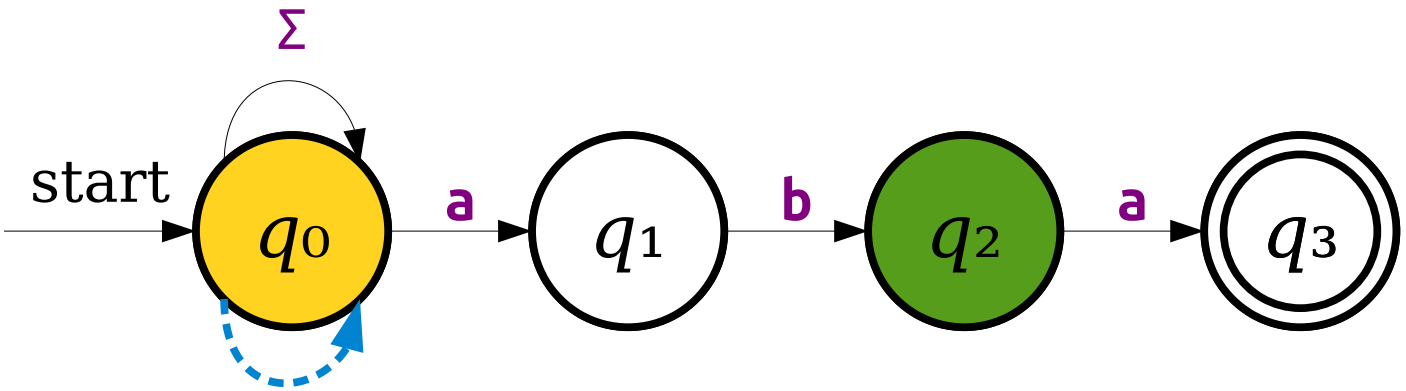

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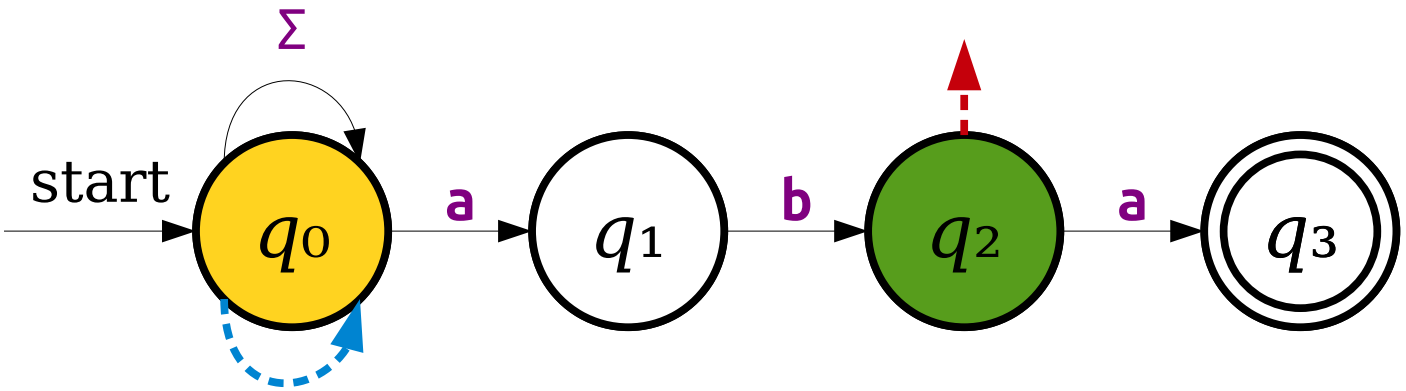

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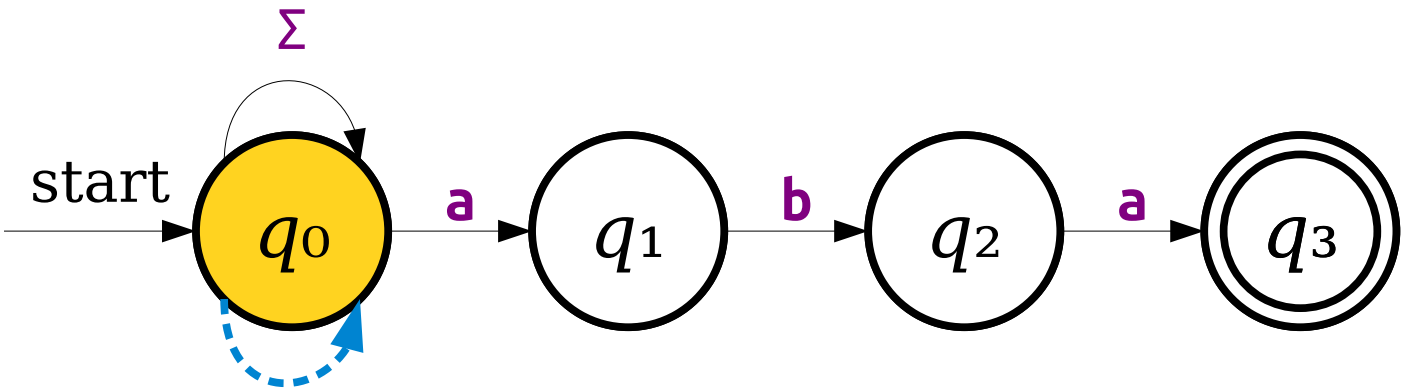

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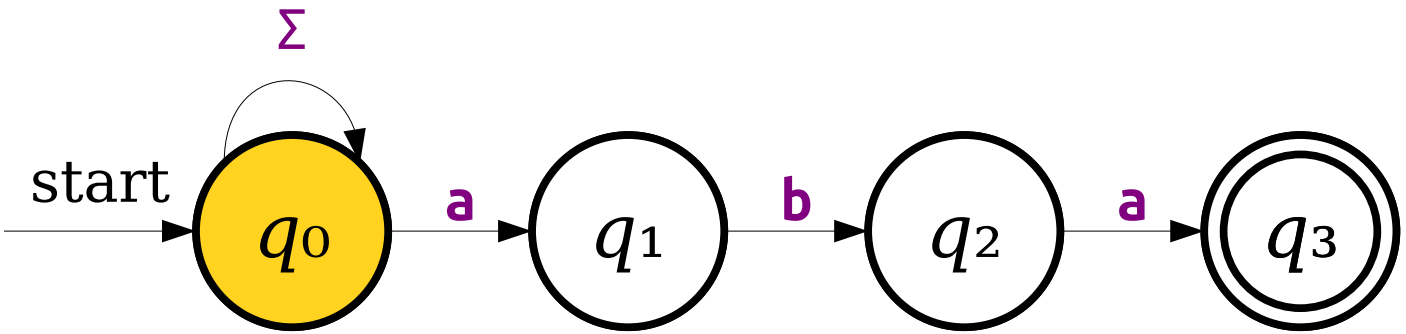

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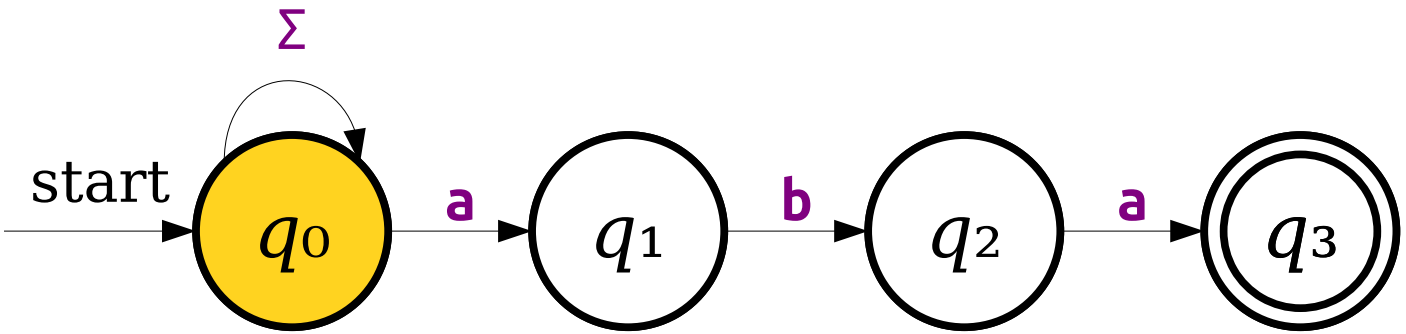

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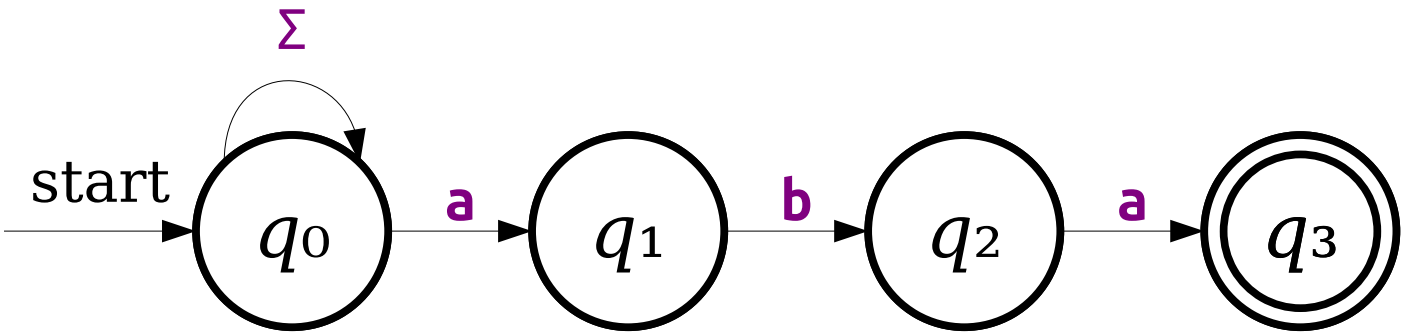

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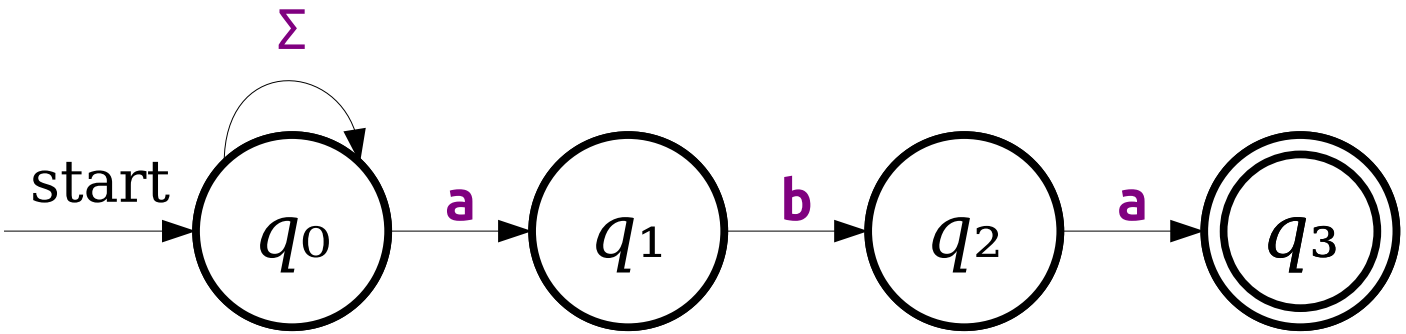

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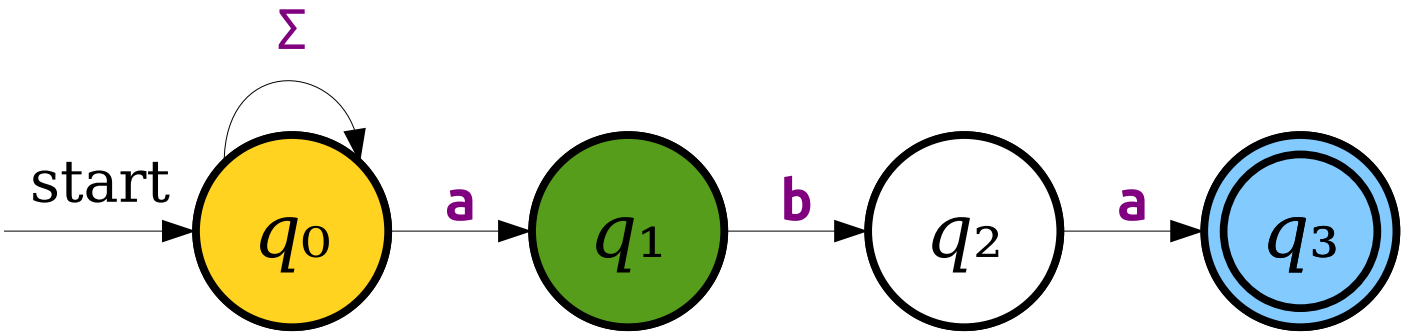

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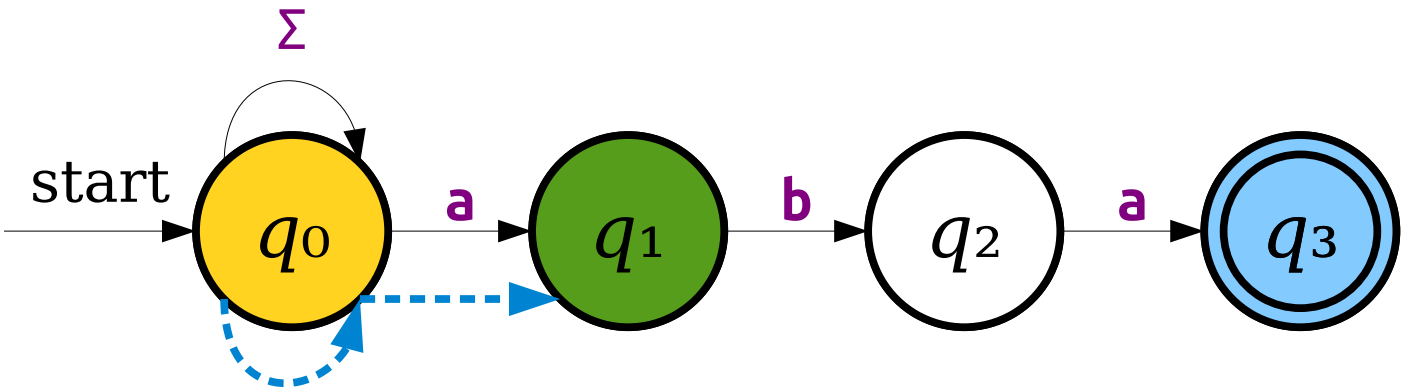

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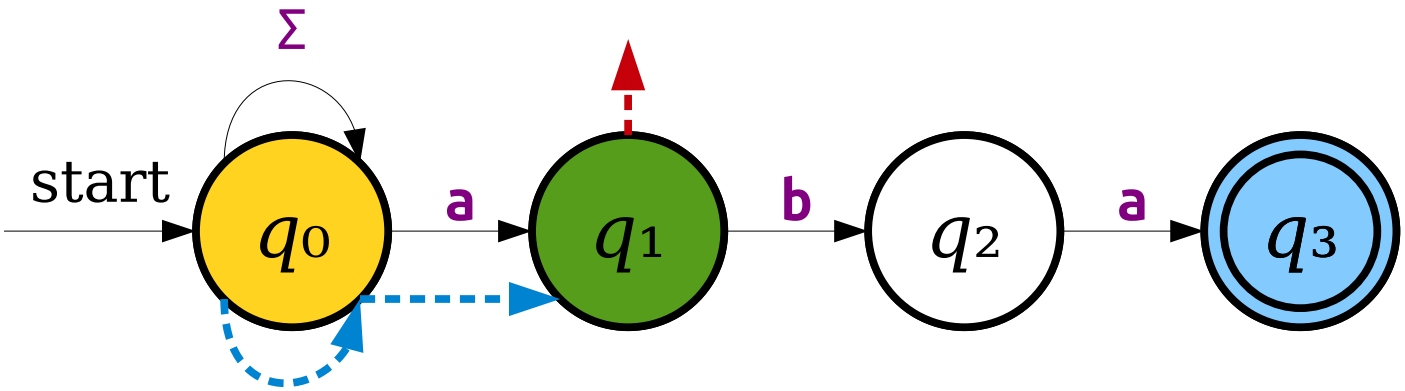

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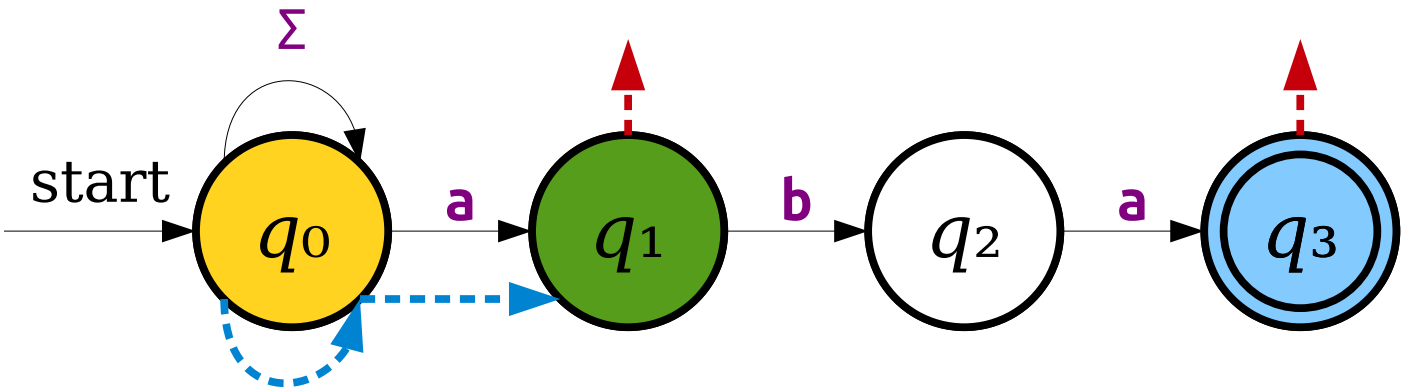

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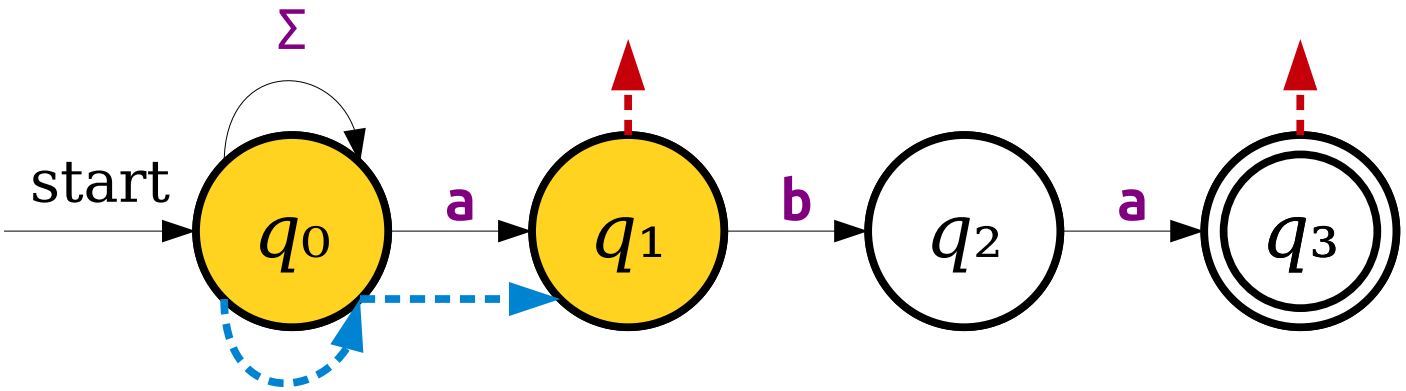

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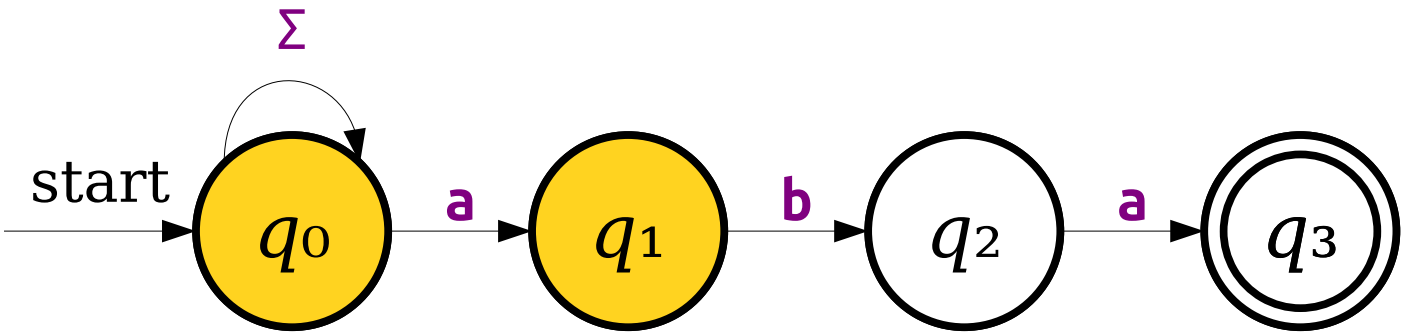

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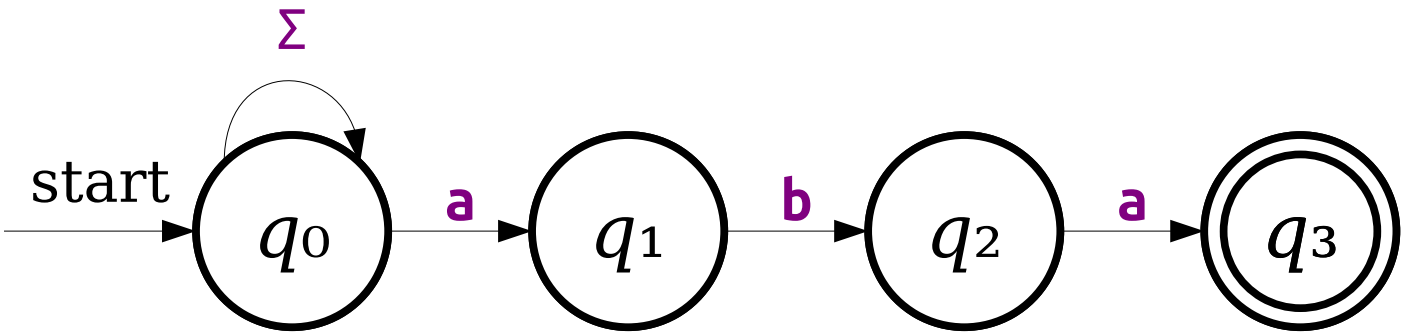

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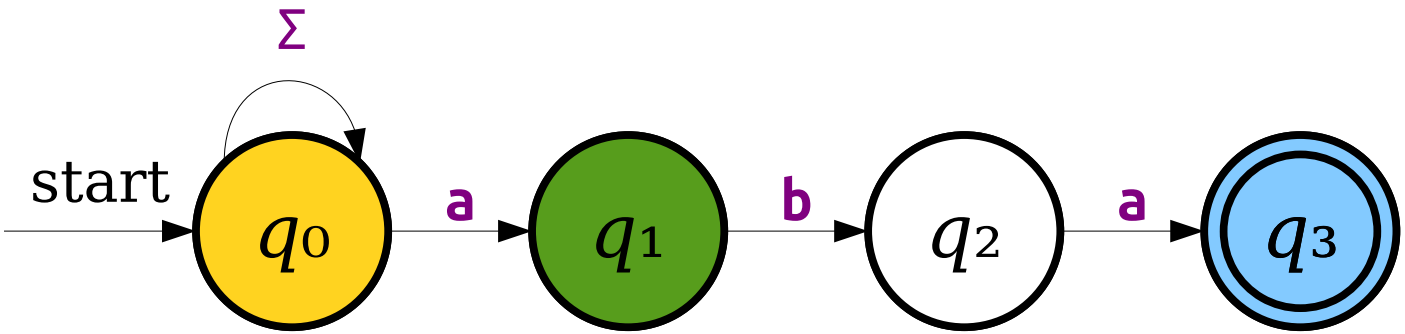

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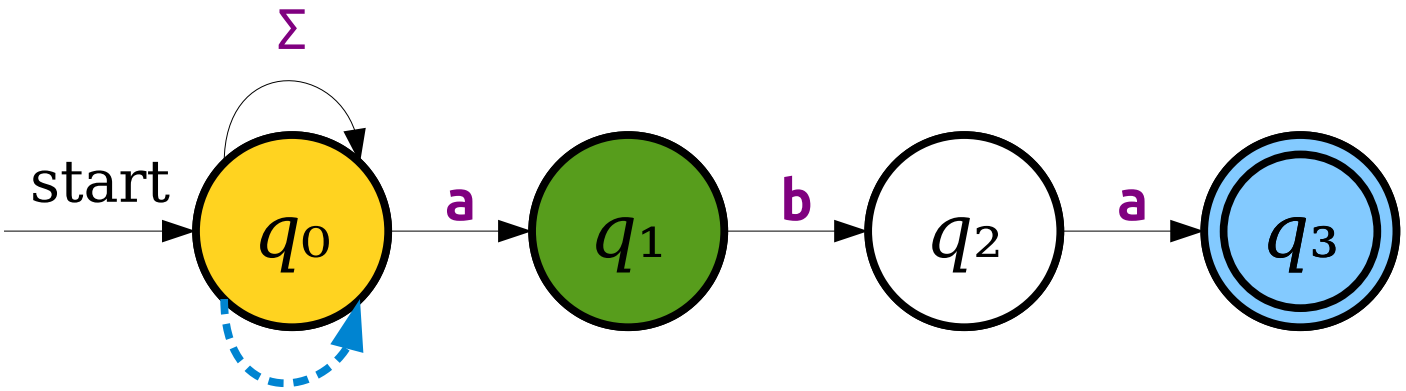

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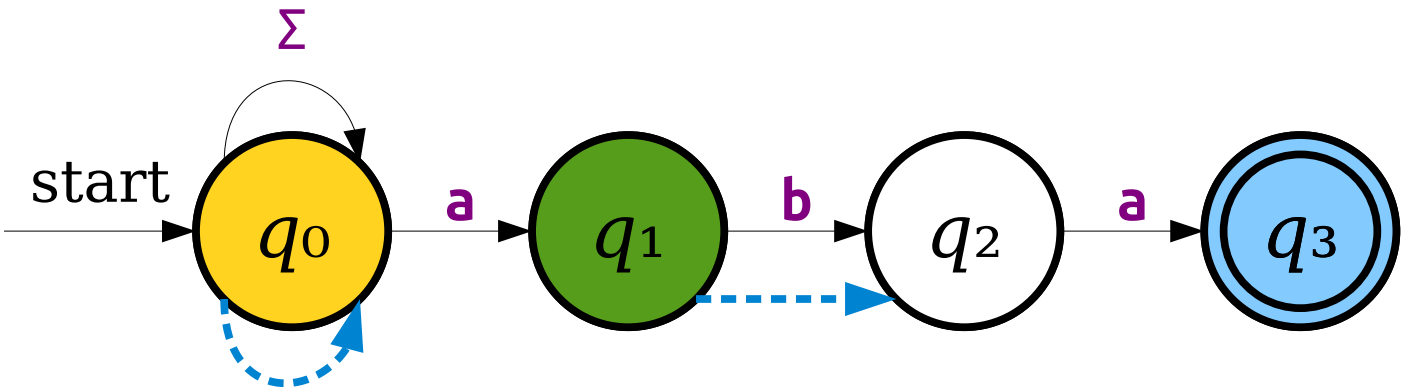
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



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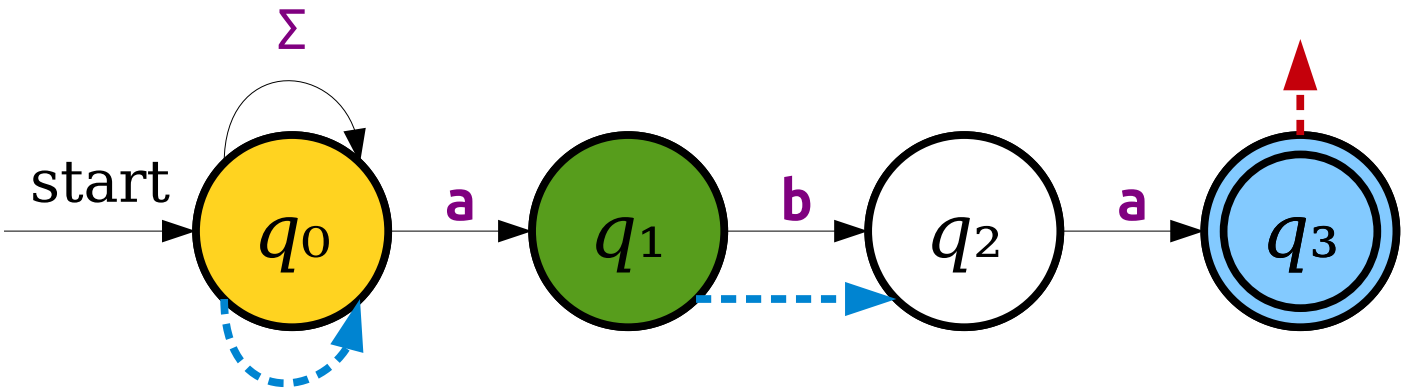
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



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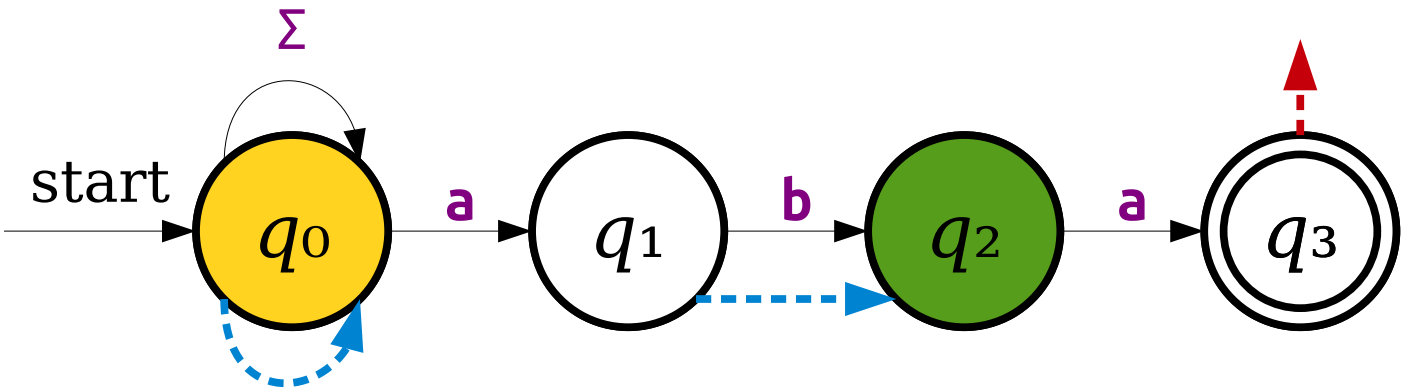

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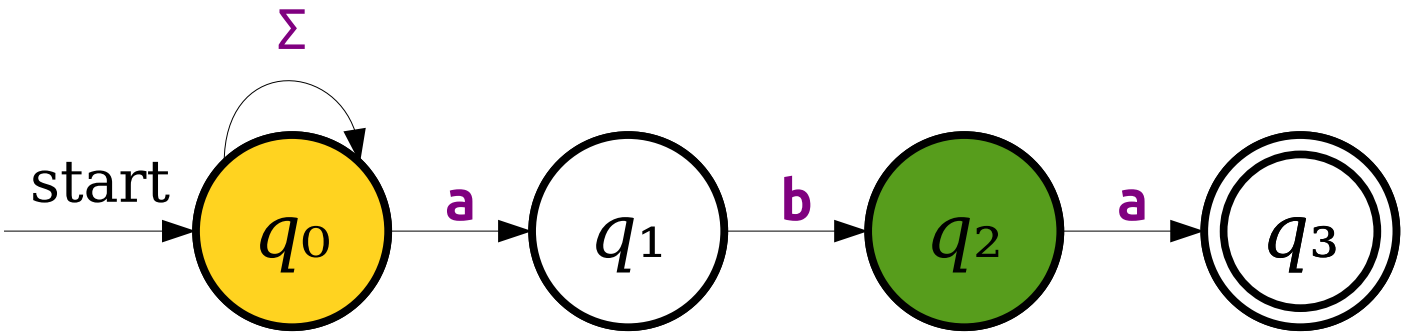

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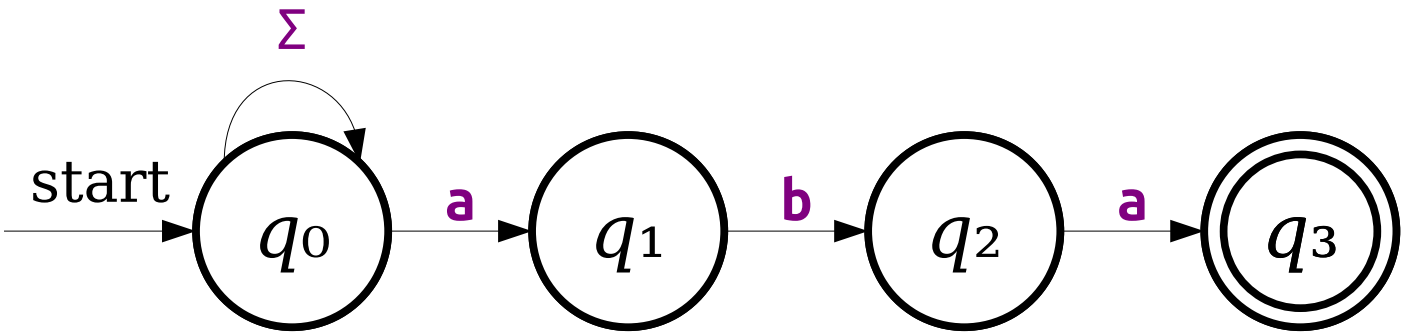

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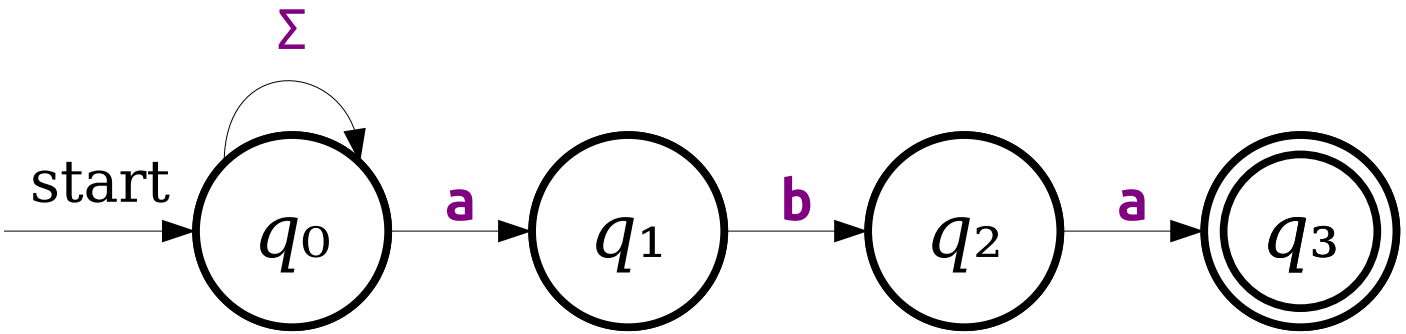

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$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
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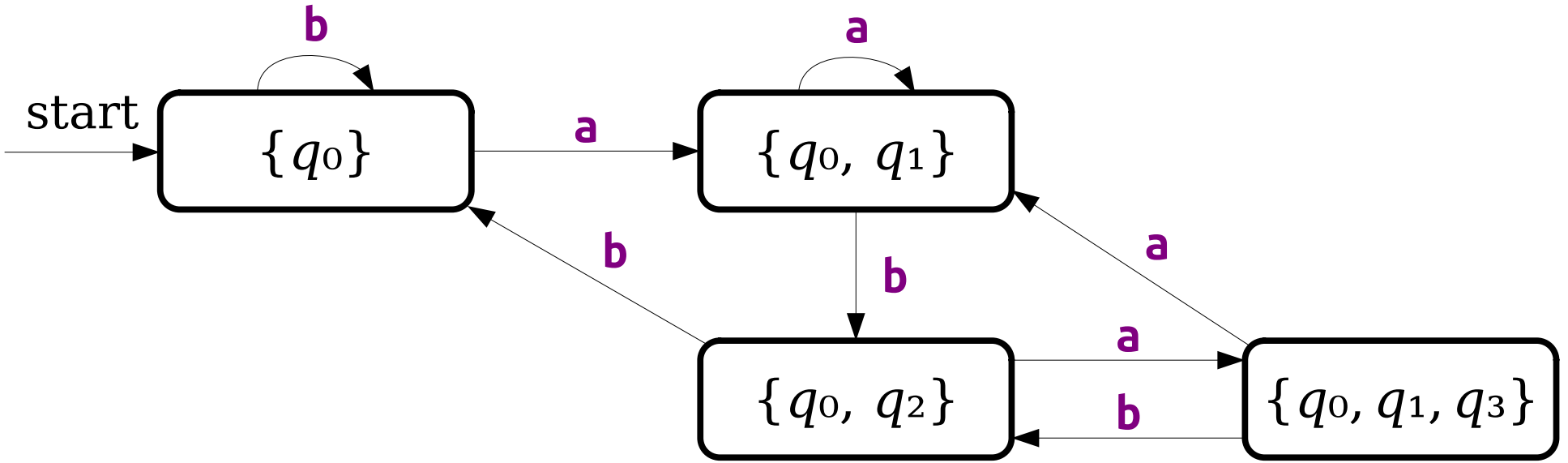

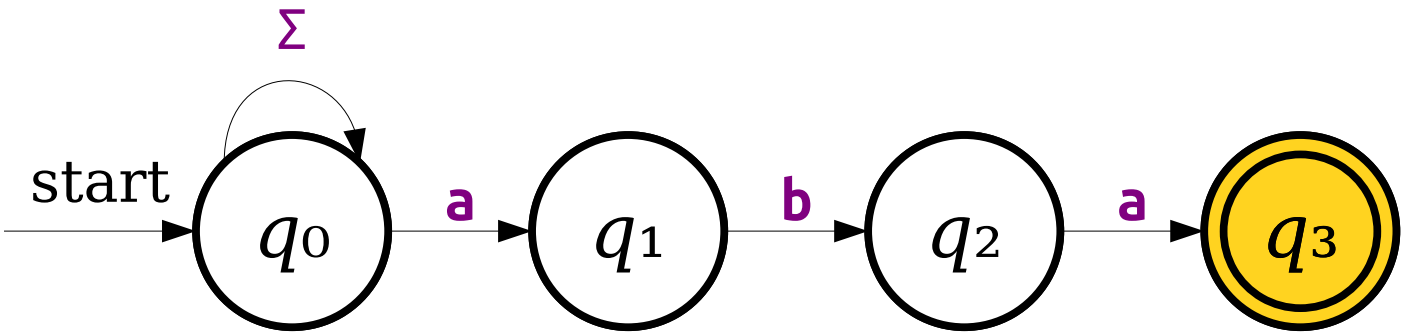

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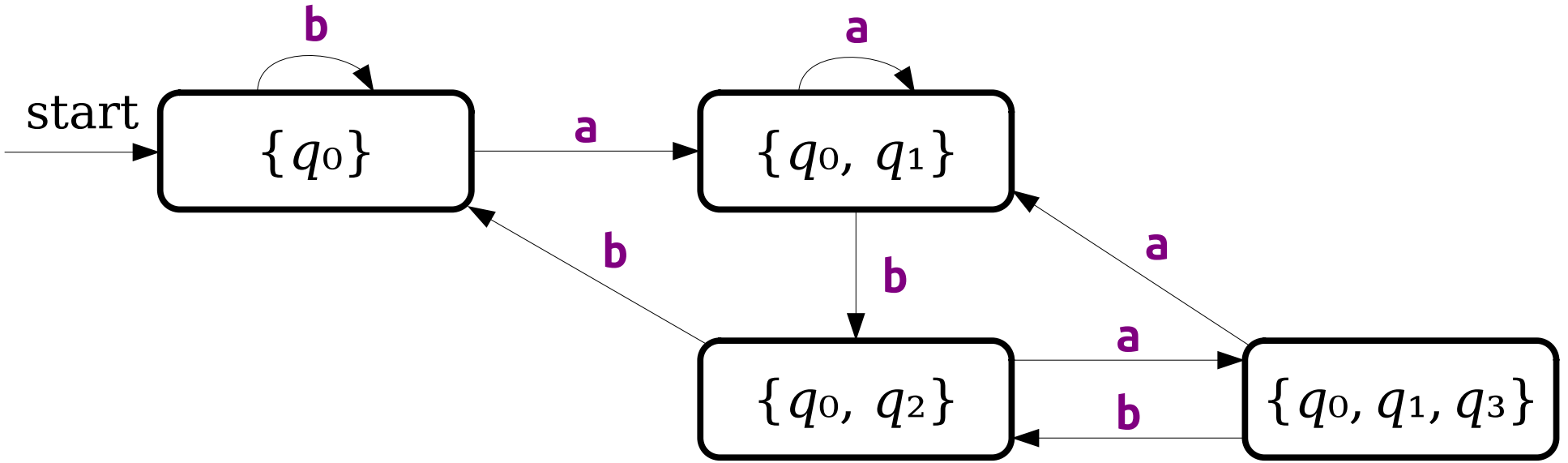

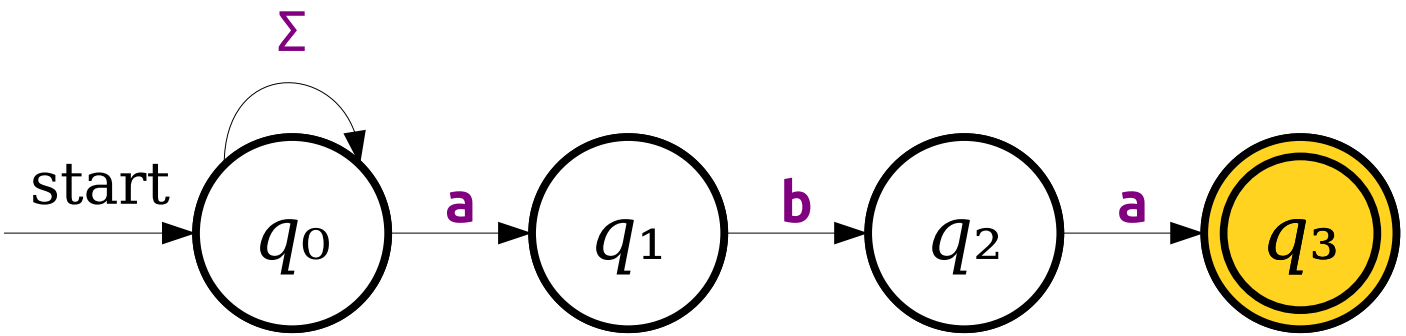

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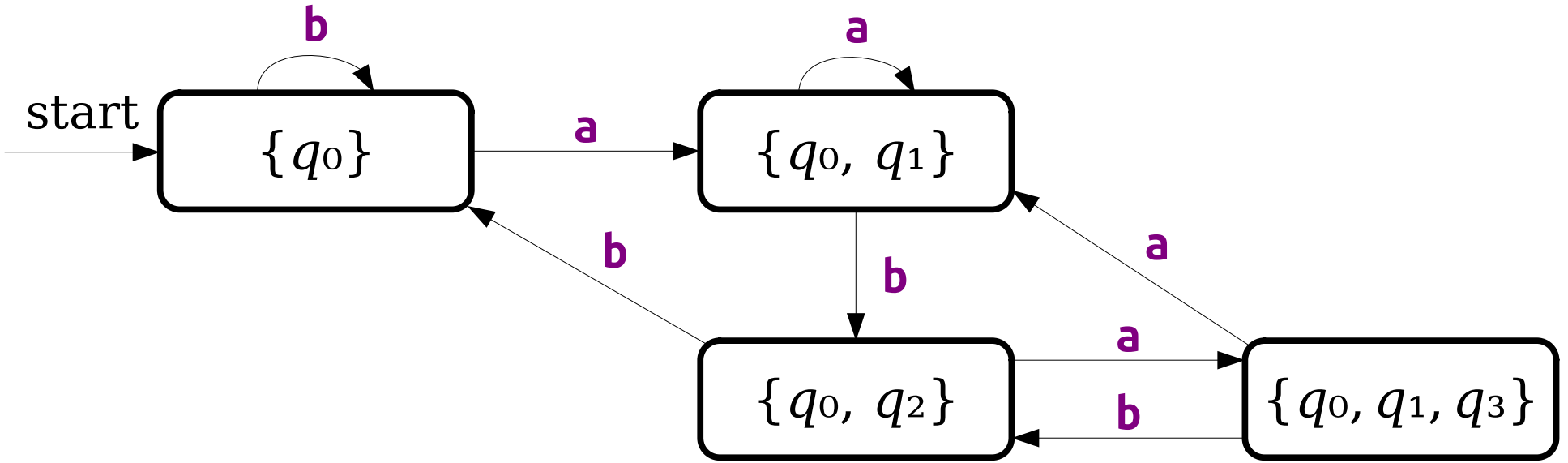

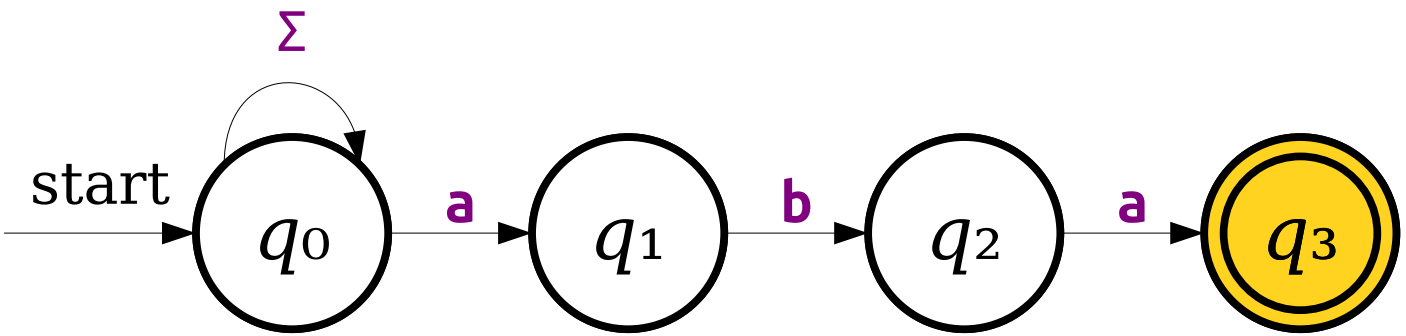

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$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
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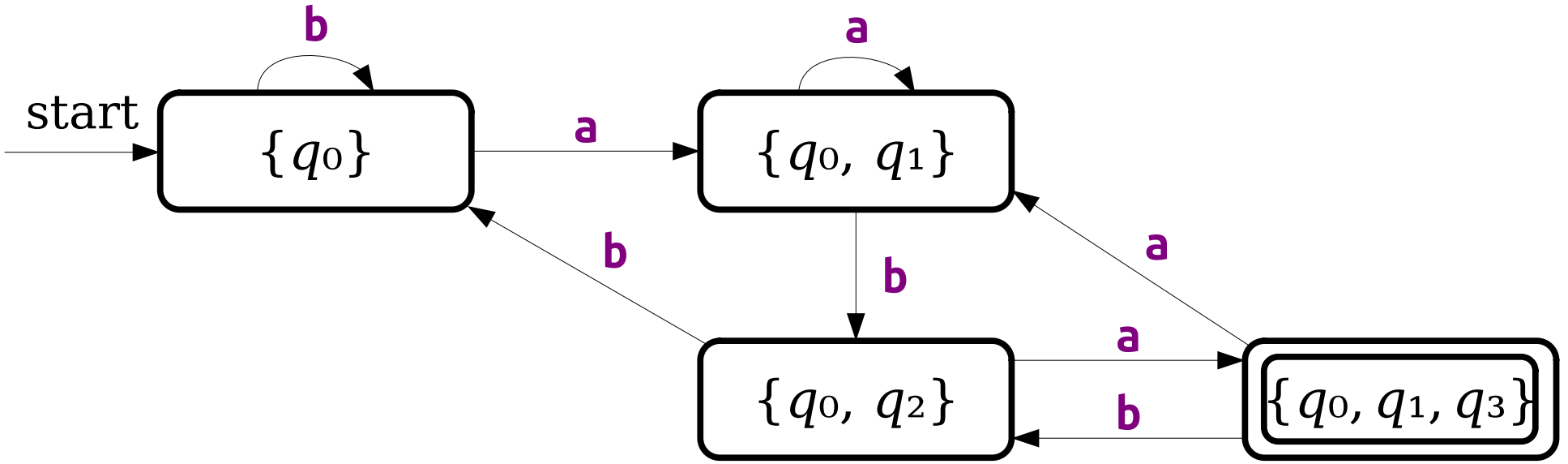

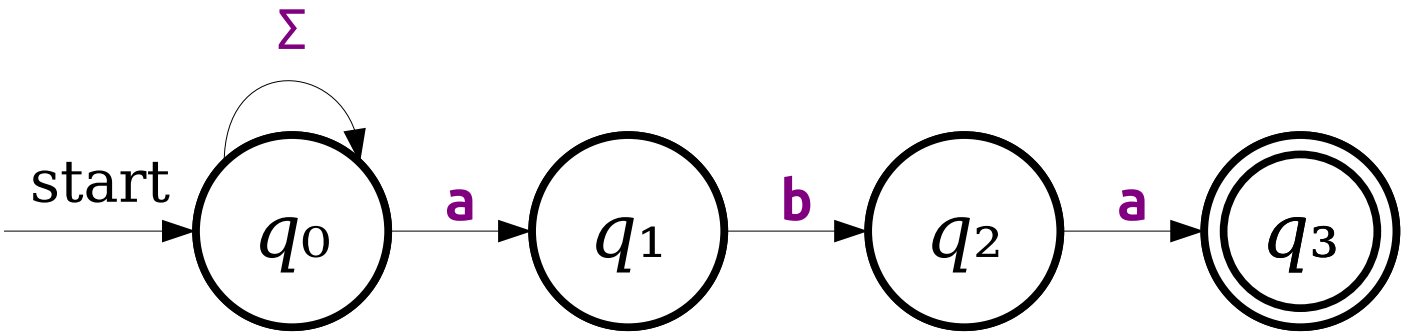

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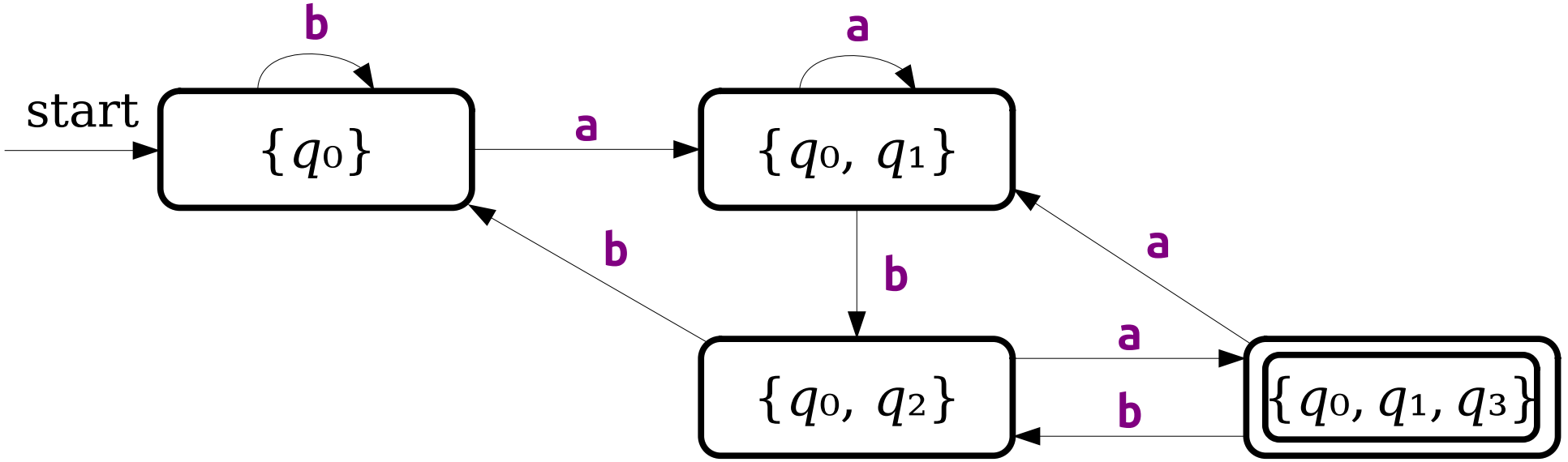

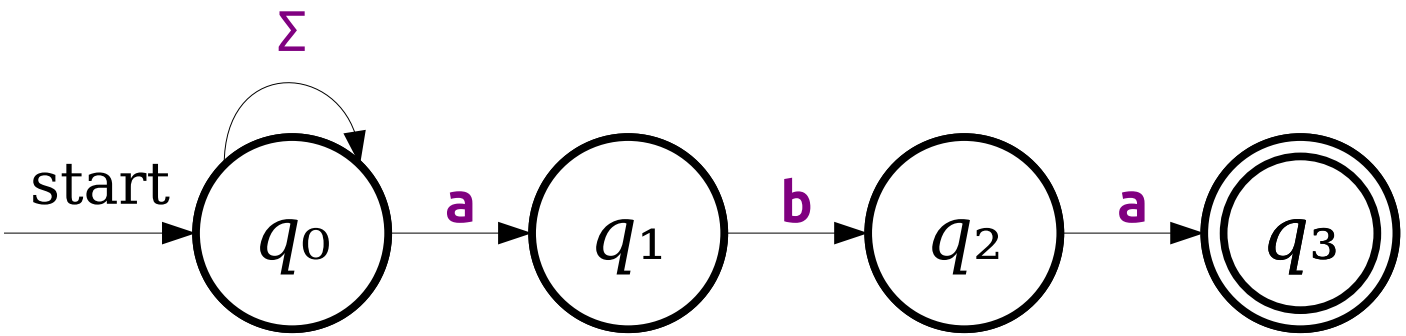

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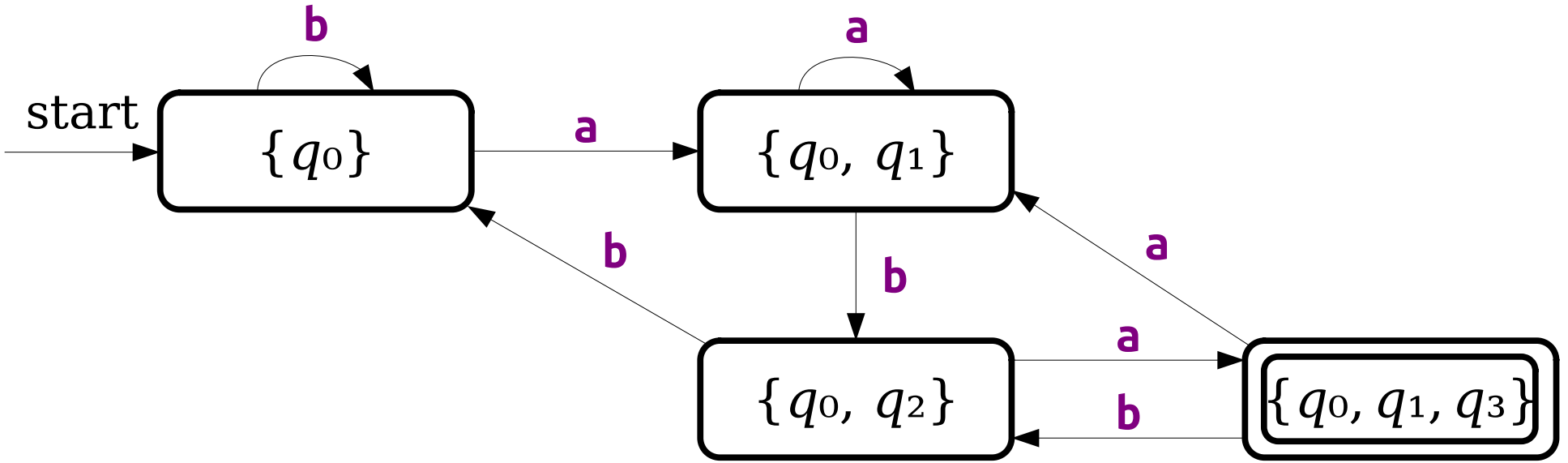

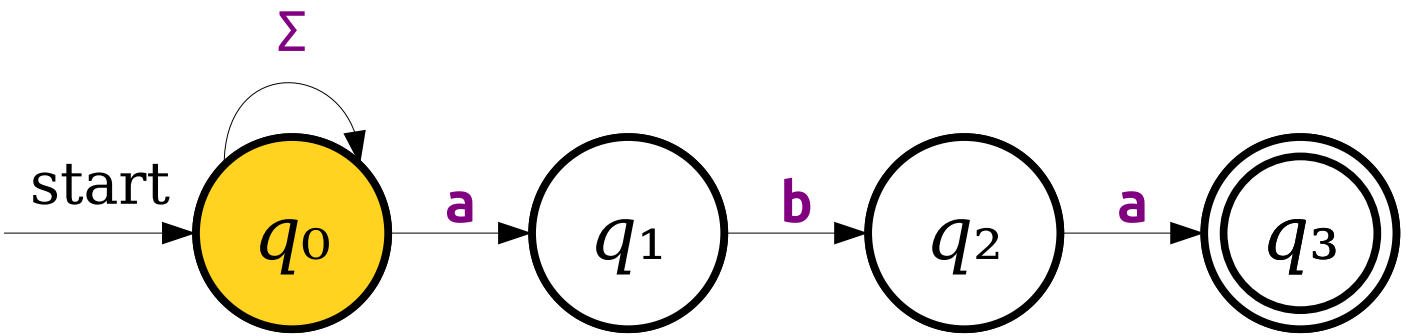

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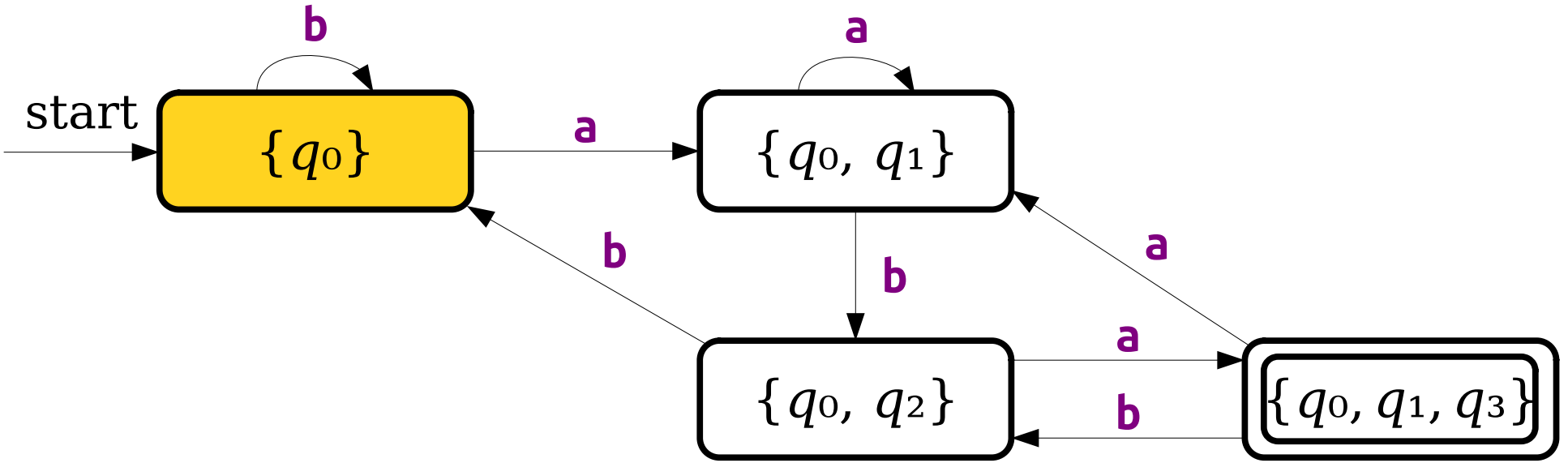

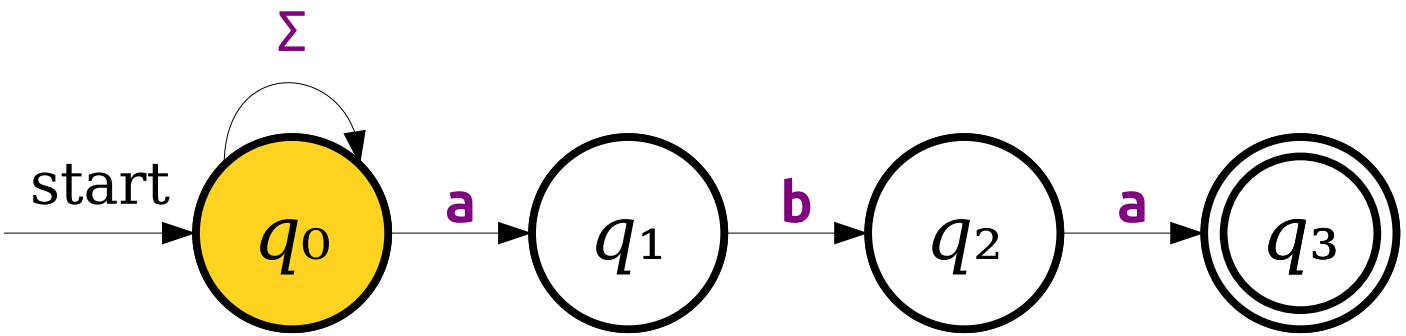
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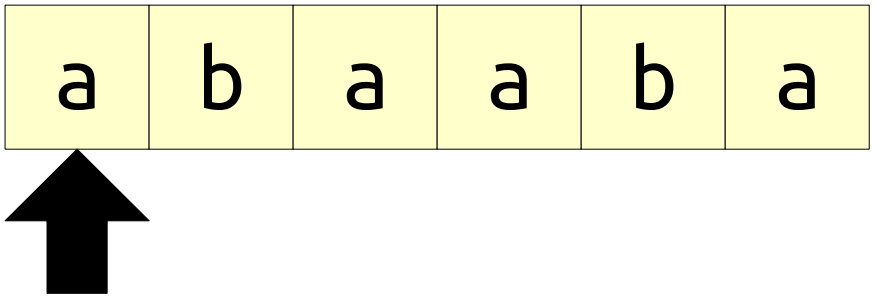
start
q_0
a
q_1
b
q_2
a
q_3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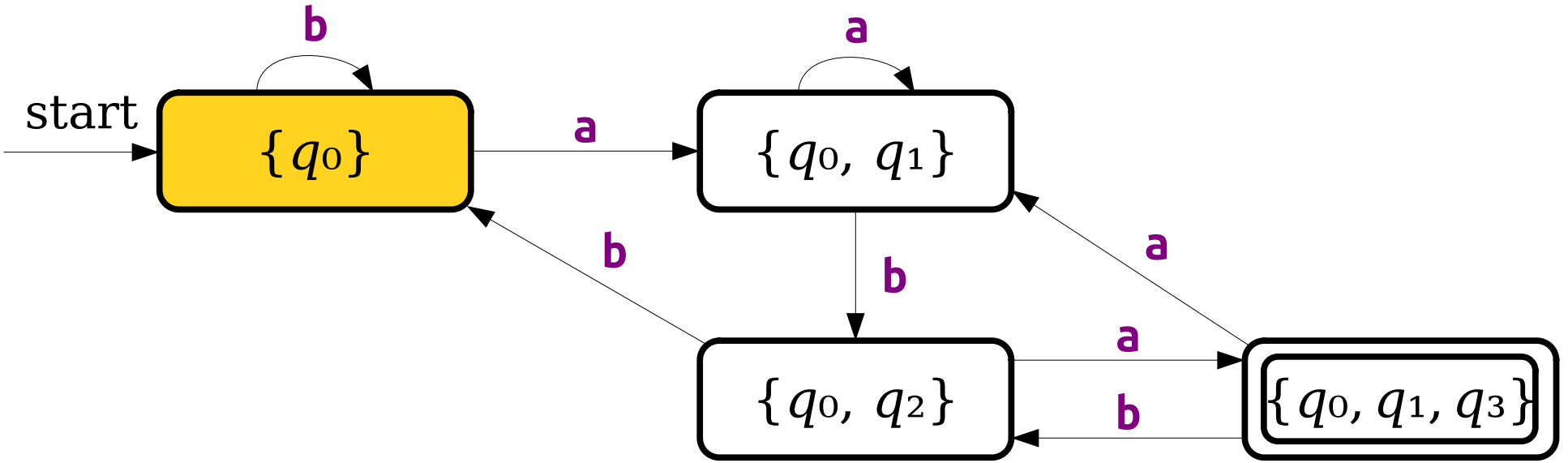
b
a
a
b
a


b
a
start
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b
a
a
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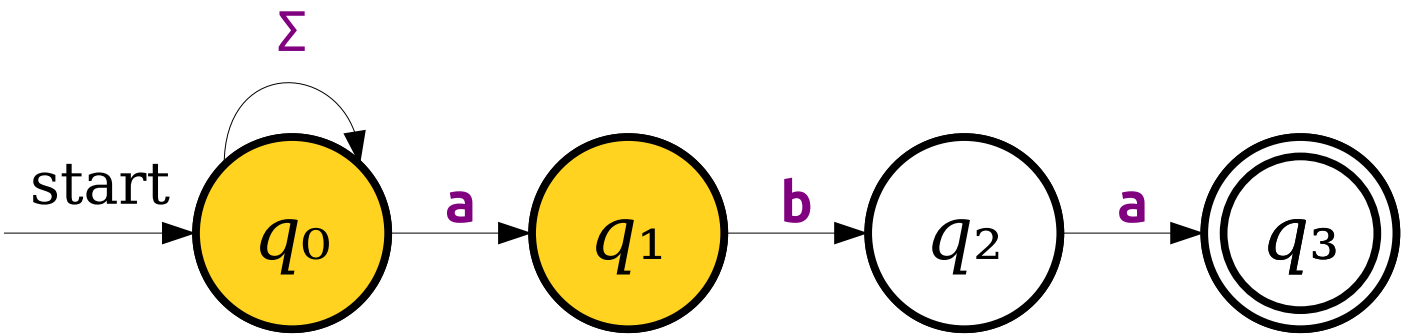
b
$\{q_0, q_1, q_3\}$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q_1
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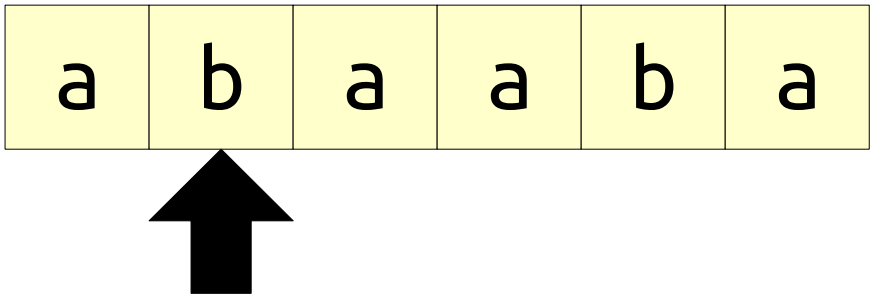
q_2
a
q_3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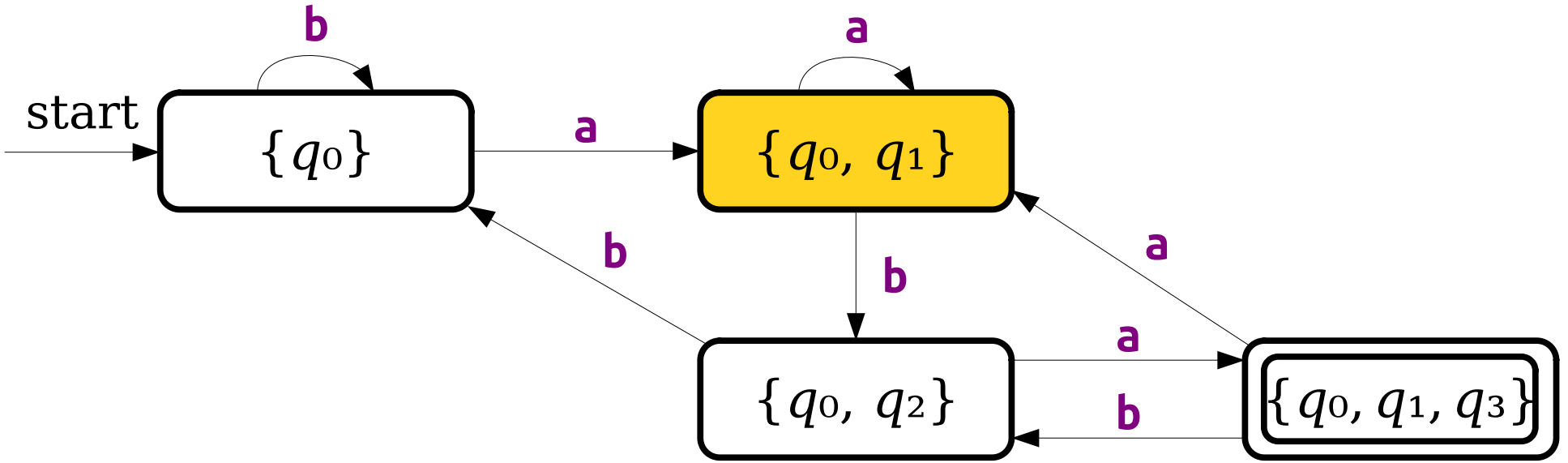
b
a
a
b
a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a
start
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b
b
a
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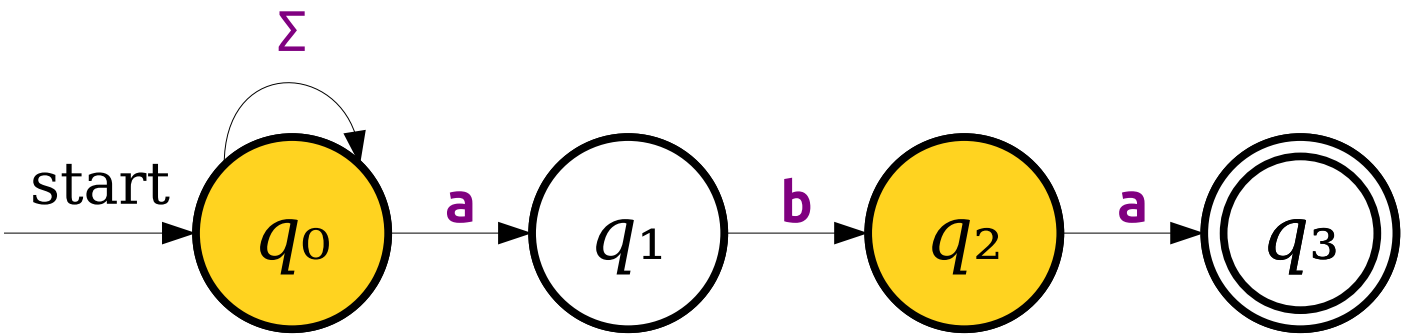
$\{q_0, q_2\}$
b
$\{q_0, q_1, q_3\}$


Σ
start
q_0
a
q_1
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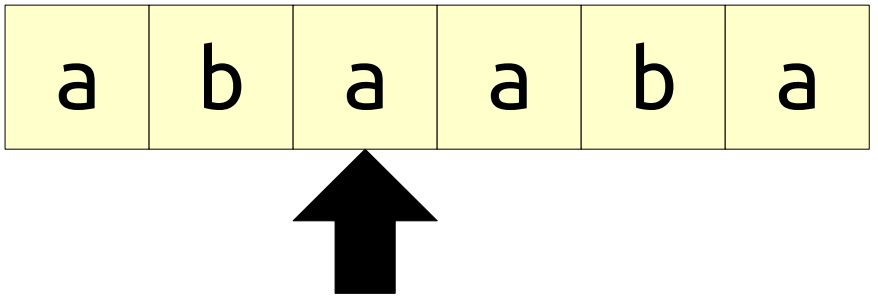
q_2
a
q_3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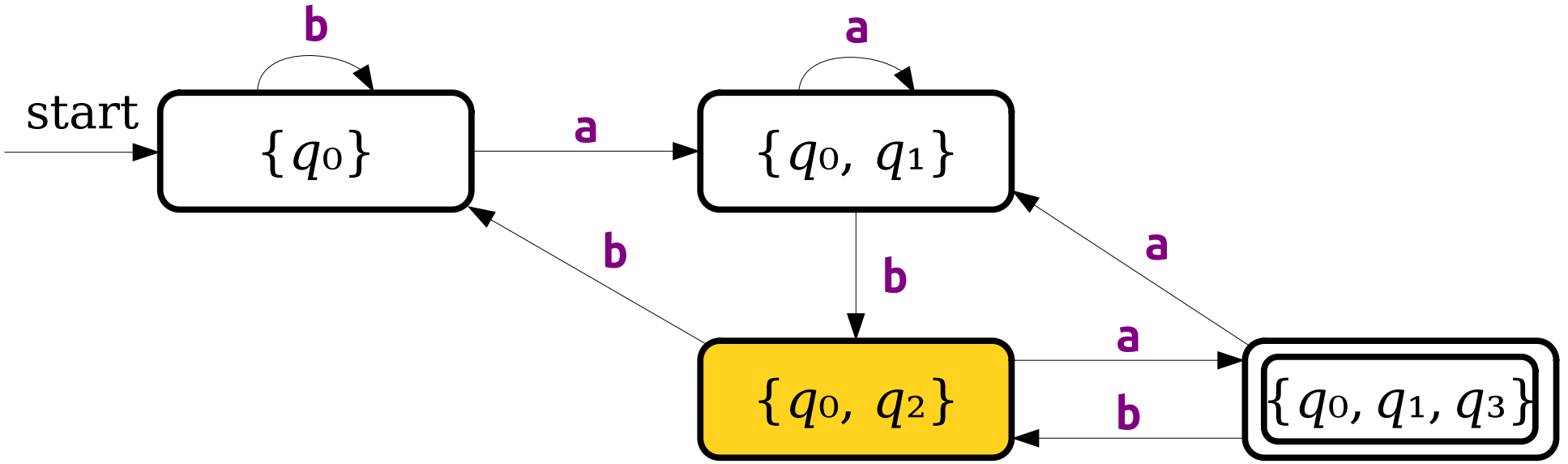
b
a
a
b
a


b
a
start
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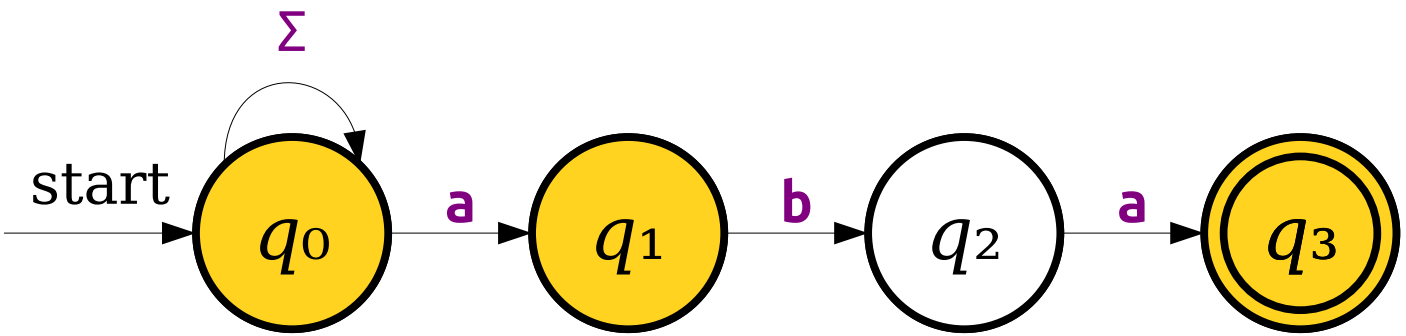
$\{q_0, q_1\}$
b
b
a
a
$\{q_0, q_2\}$
b
$\{q_0, q_1, q_3\}$


Σ
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q_0
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q_2
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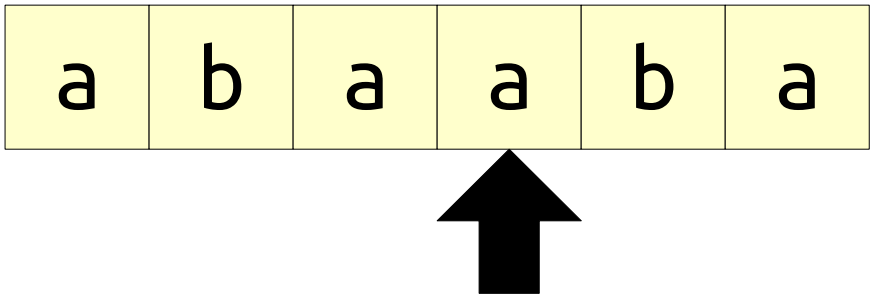
q_3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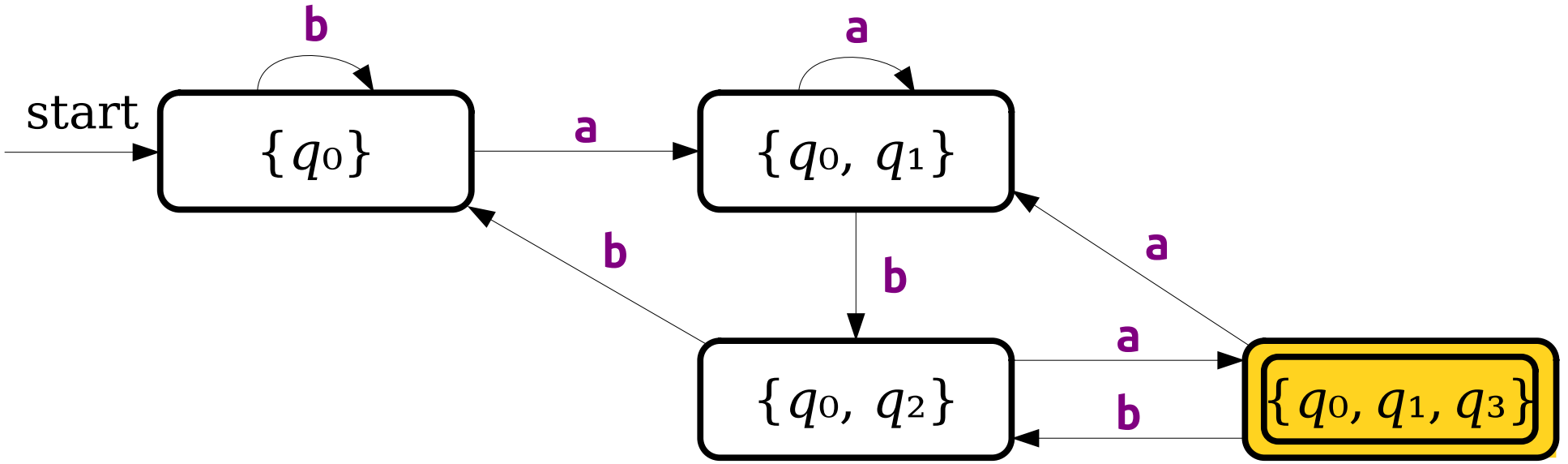
a
a
b
a


b
a
start
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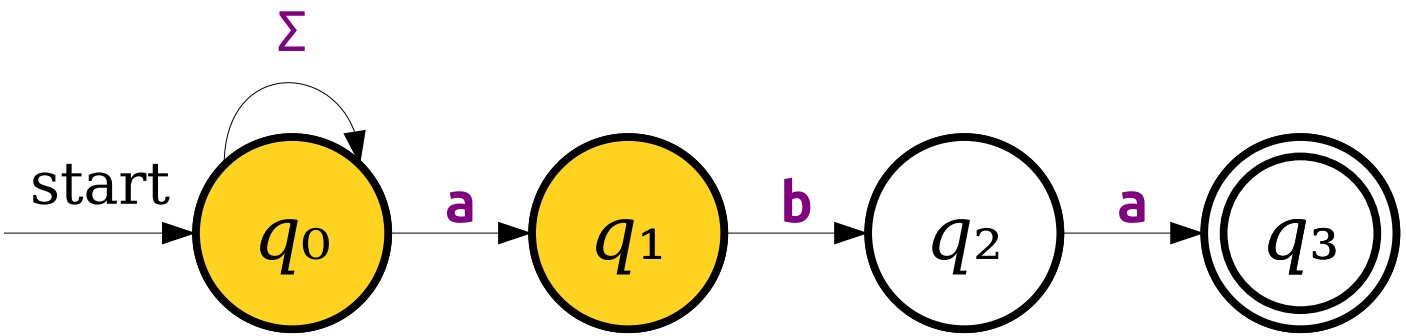
$\{q_0, q_1\}$
b
b
a
a
$\{q_0, q_2\}$
b
$\{q_0, q_1, q_3\}$


Σ
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q_0
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q_1
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q_2
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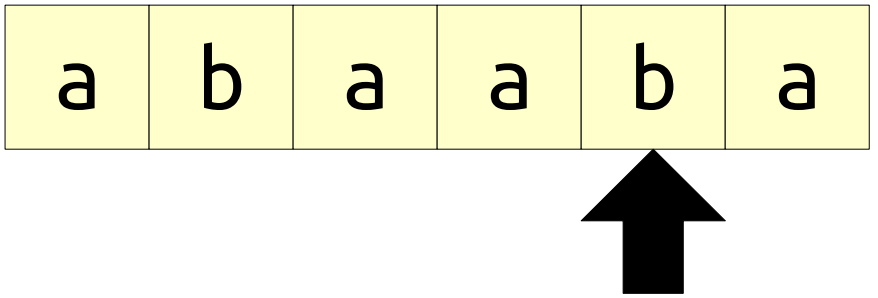
q_3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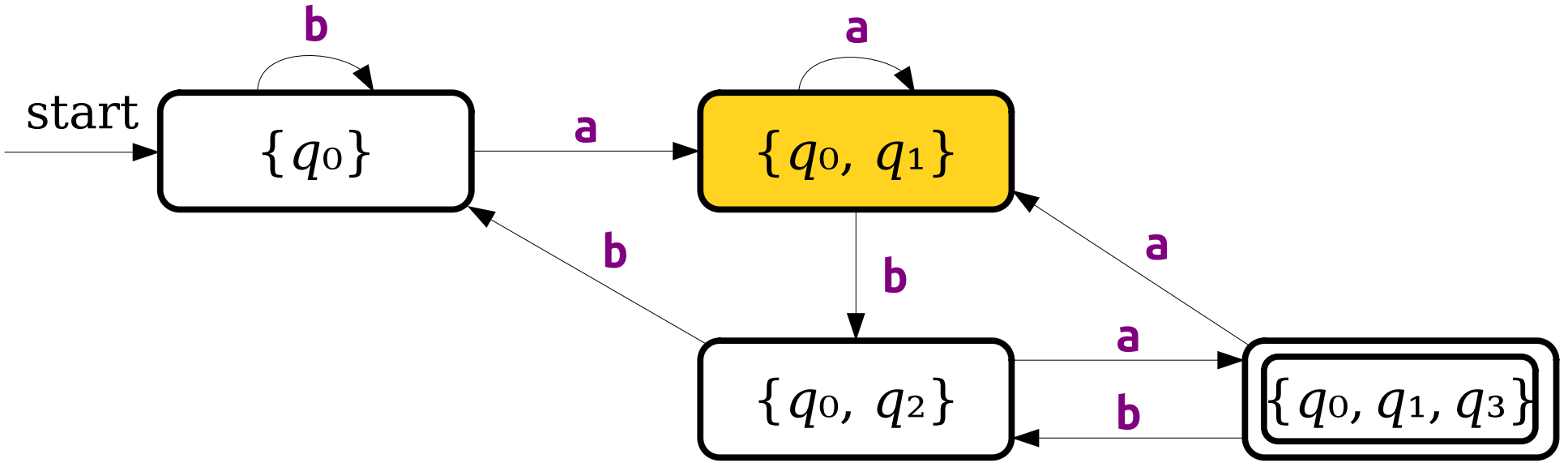
a
a
b
a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a
start
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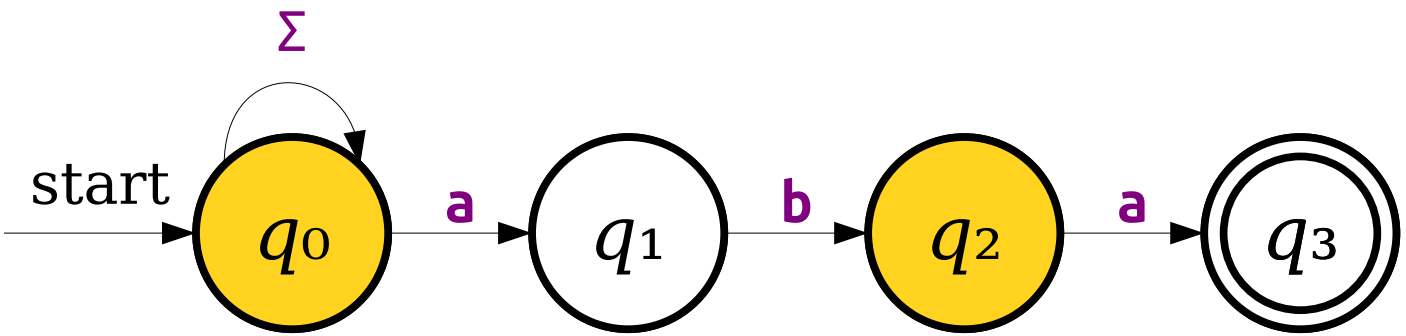
b
a
a
$\{q_0, q_2\}$
b
$\{q_0, q_1, q_3\}$


Σ
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q_0
a
q_1
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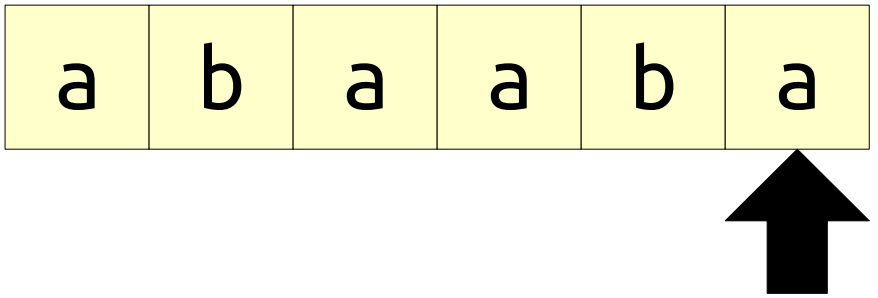
q_2
a
q_3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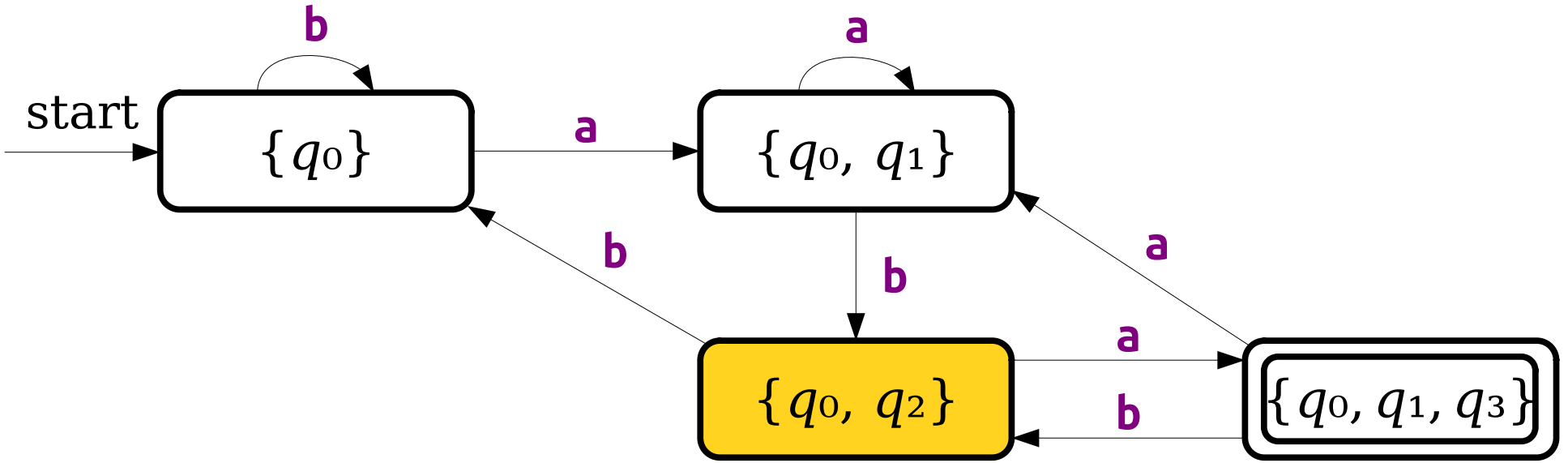
b
a
a
b
a


b
a
start
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b
b
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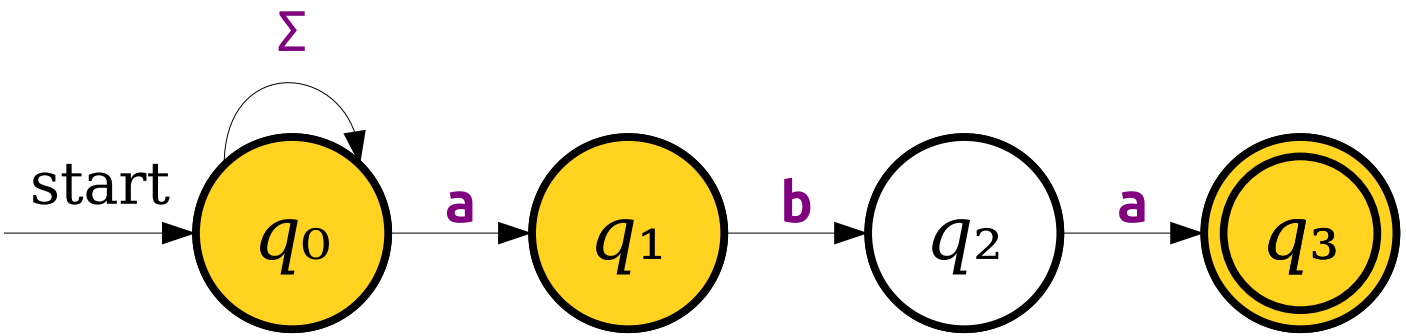
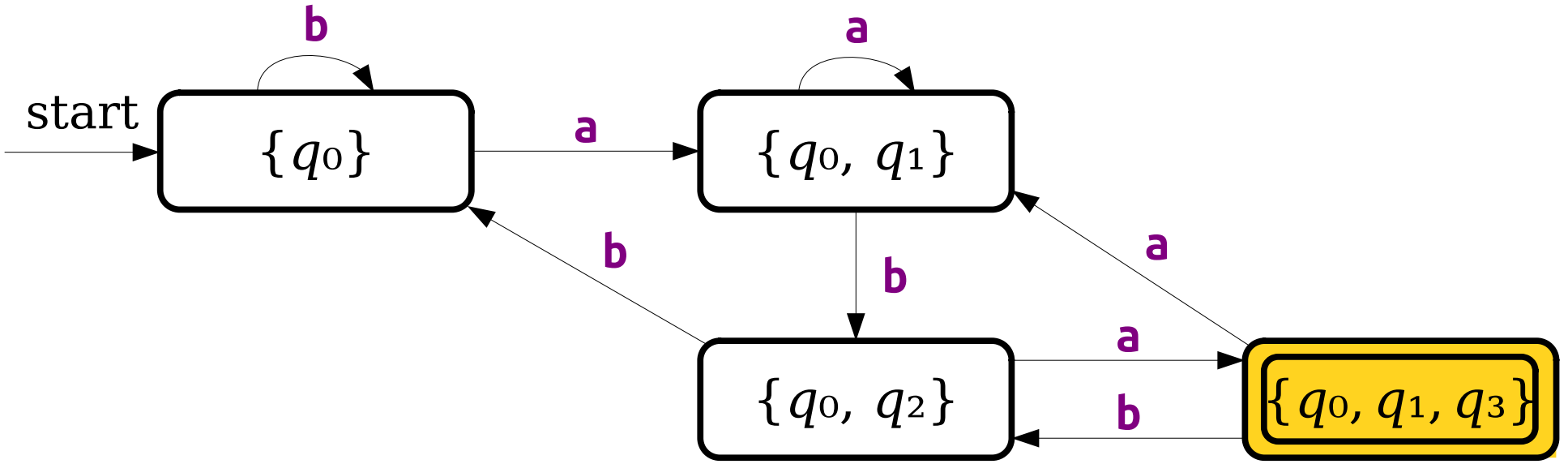
a
$\{q_0, q_2\}$
b
$\{q_0, q_1, q_3\}$



a	b	a	a	b	a



The Subset Construction

- This procedure for turning an NFA for a language L into a DFA for a language L is called the ***subset construction***.
  - It's sometimes called the ***powerset construction***; it's different names for the same thing!
- Intuitively:
  - Each state in the DFA corresponds to a set of states from the NFA.
  - Each transition in the DFA corresponds to what transitions would be taken in the NFA when using the massive parallel intuition.
  - The accepting states in the DFA correspond to which sets of states would be considered accepting in the NFA when using the massive parallel intuition.

The Subset Construction

- In converting an NFA to a DFA, the DFA's states correspond to sets of NFA states.
- ***Useful fact:*** $|\wp(S)| = 2^{|S|}$ for any finite set S.
- In the worst-case, the construction can result in a DFA that is *exponentially larger* than the original NFA.
- ***Question to ponder:*** Can you find a family of languages that have NFAs of size n, but no DFAs of size less than 2^n?

A language L is called a ***regular language*** if there exists a DFA D such that $\mathscr{L}(D) = L$.

An Important Result

Theorem: A language L is regular if and only if there is some NFA N such that $\mathscr{L}(N) = L$.

Proof Sketch: Pick a language L. First, assume L is regular. That means there's a DFA D where $\mathscr{L}(D) = L$. Every DFA is "basically" an NFA, so there's an NFA (D) whose language is L.

Next, assume there's an NFA N such that $\mathscr{L}(N) = L$. Using the subset construction, we can build a DFA D where $\mathscr{L}(N) = \mathscr{L}(D)$. Then we have that $\mathscr{L}(D) = L$, so L is regular. ■-*ish*

Why This Matters

- We now have two perspectives on regular languages:
  - Regular languages are languages accepted by DFAs.
  - Regular languages are languages accepted by NFAs.
- We can now reason about the regular languages in two different ways.

Properties of Regular Languages

The Complement of a Language

- Given a language $L \subseteq \Sigma^*$, the ***complement*** of that language (denoted $\overline{L}$) is the language of all strings in Σ^* that aren't in L.
- Formally:

$$\overline{L} = \Sigma^* - L$$

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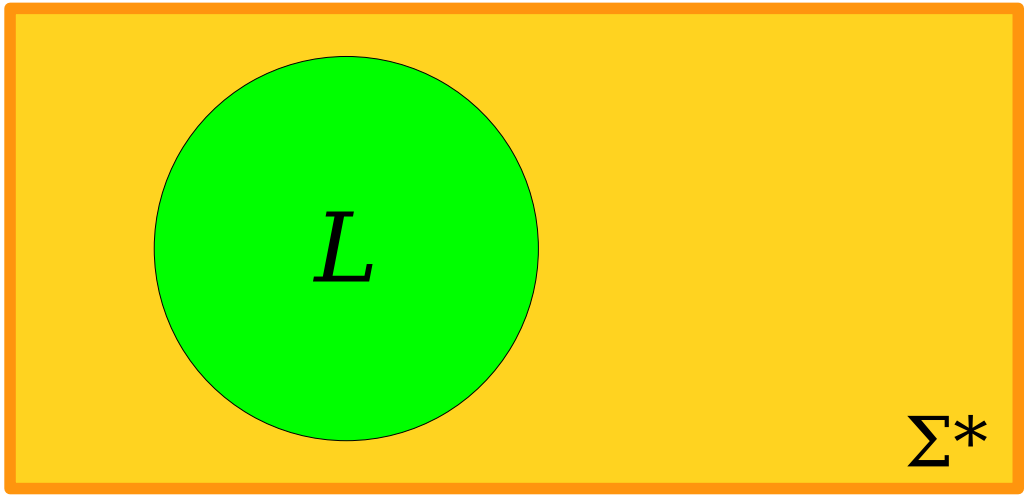

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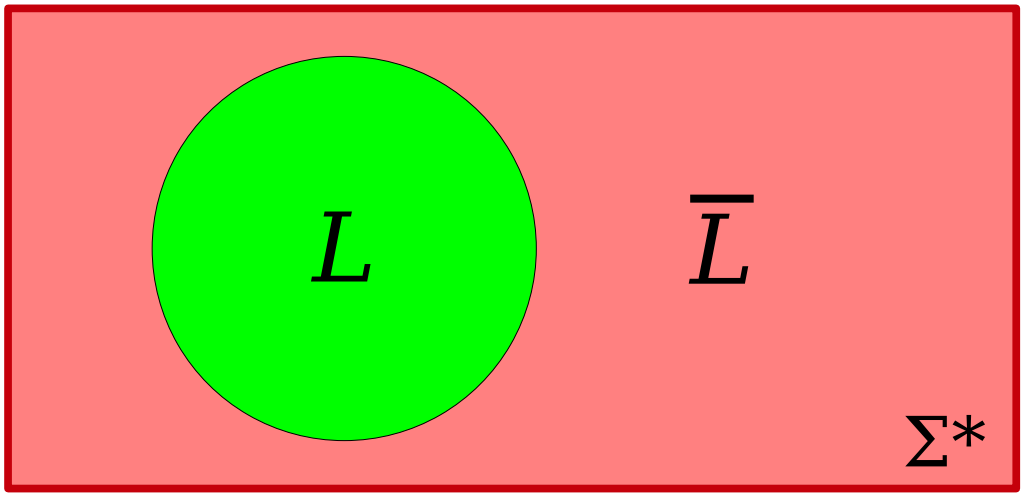

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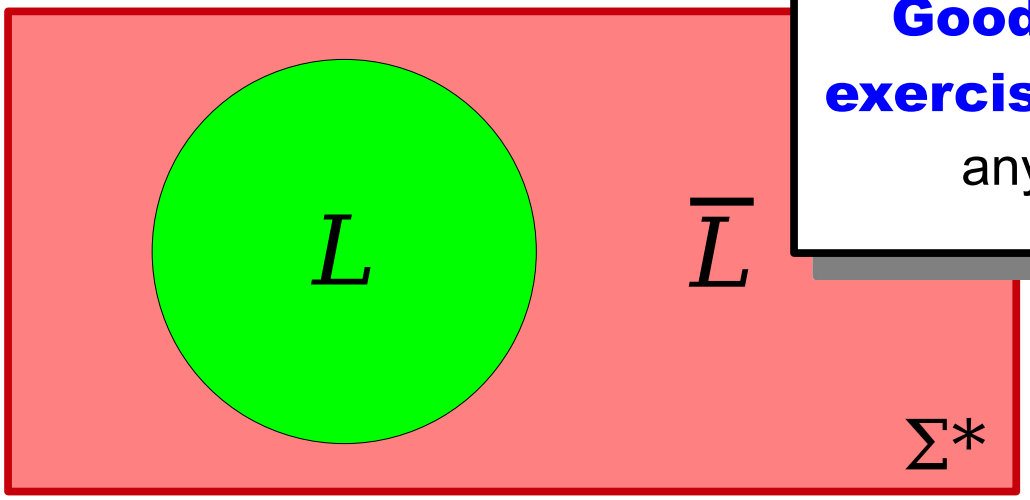
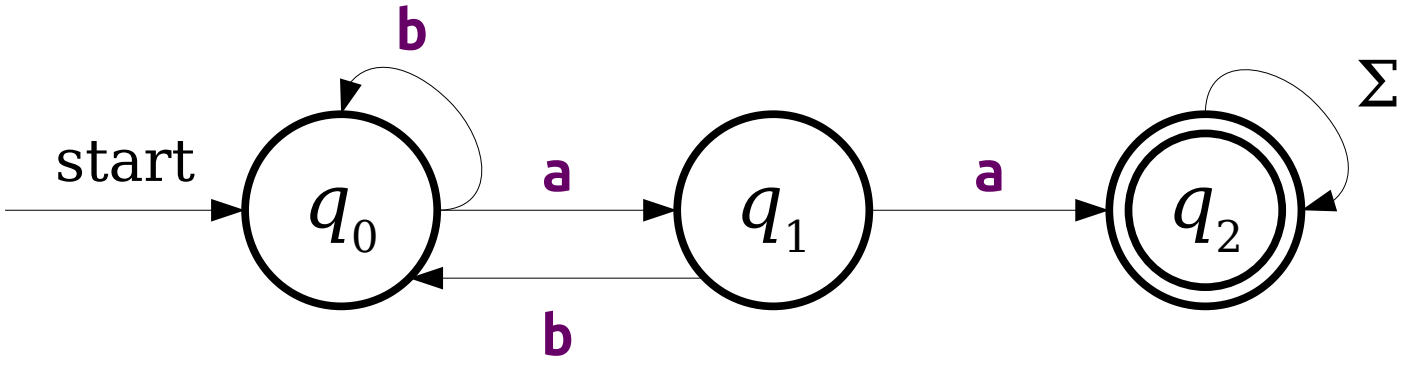


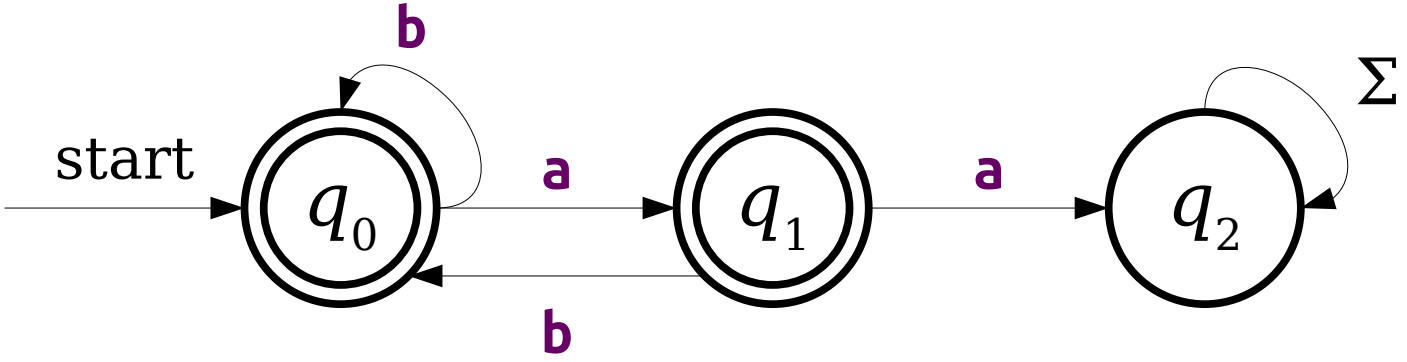
Good proofwriting exercise: prove $\overline{\overline{L}} = L$ for any language L.

Complementing Regular Languages

$L = \{ w \in \{\texttt{a}, \texttt{b}\}^* \mid w \text{ contains } \texttt{aa} \text{ as a substring} \}$

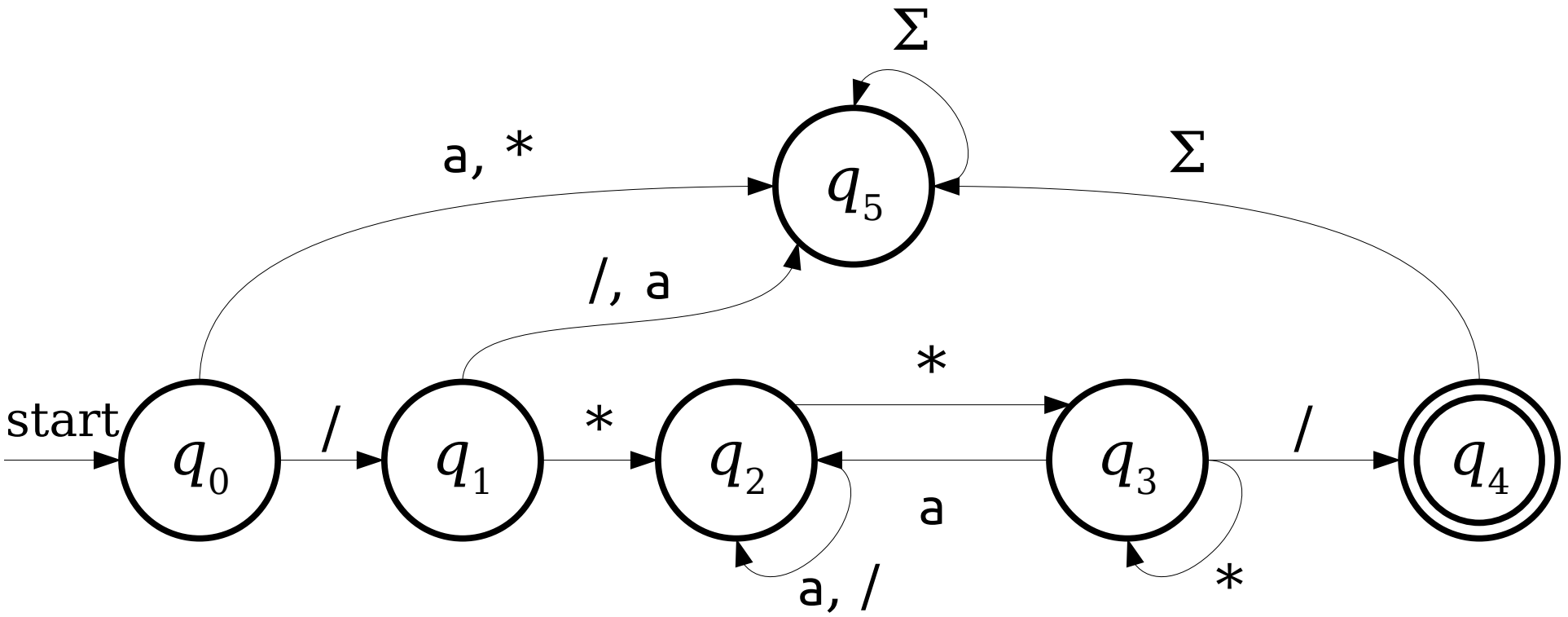


$\overline{L} = \{ w \in \{\texttt{a}, \texttt{b}\}^* \mid w \textbf{\textit{ does not }} \text{contain } \texttt{aa} \text{ as a substring} \}$



Complementing Regular Languages

$L = \{\ w \in \{\texttt{a}, \texttt{*}, \texttt{/}\}^* \mid w \text{ represents a C-style comment}\ \}$



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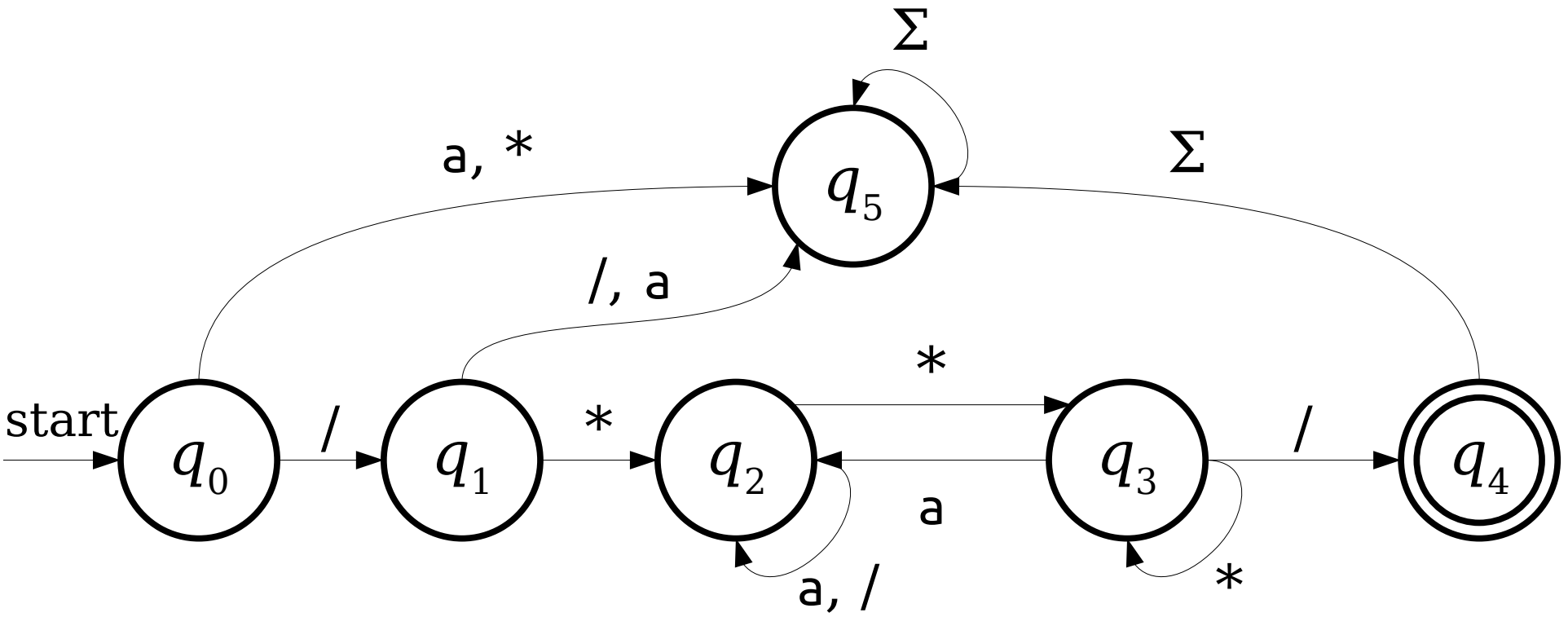

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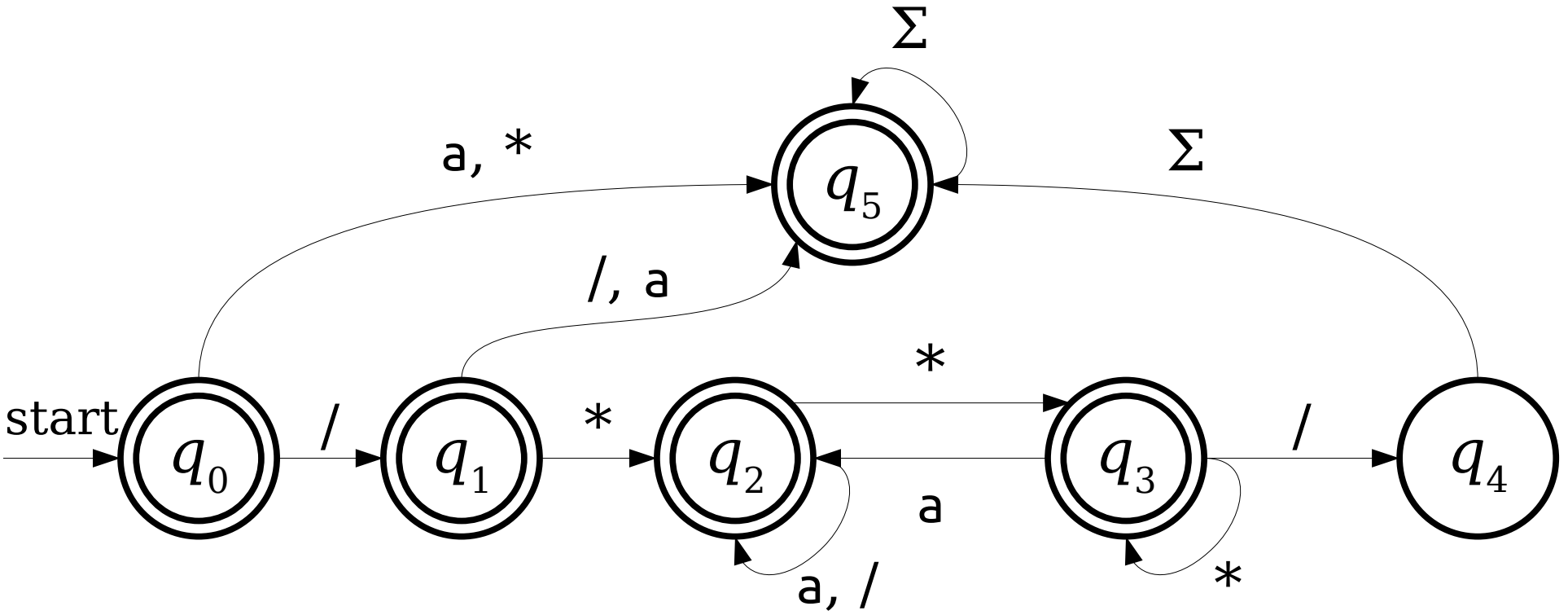
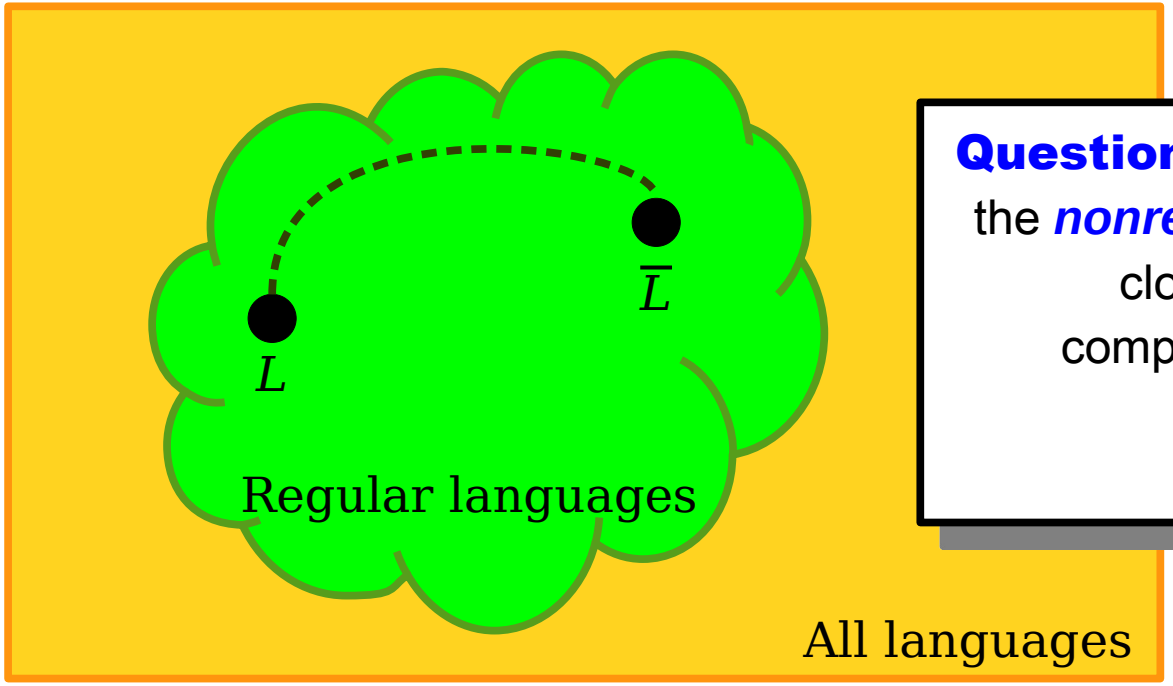


Closure Properties

- ***Theorem:*** If L is a regular language, then $\overline{L}$ is also a regular language.
- As a result, we say that the regular languages are ***closed under complementation***.



Question to ponder: are the ***nonregular*** languages closed under complementation?

The Union of Two Languages

- If L_1 and L_2 are languages over the alphabet Σ, the language $L_1 \cup L_2$ is the language of all strings in at least one of the two languages.
- If L_1 and L_2 are regular languages, is $L_1 \cup L_2$?

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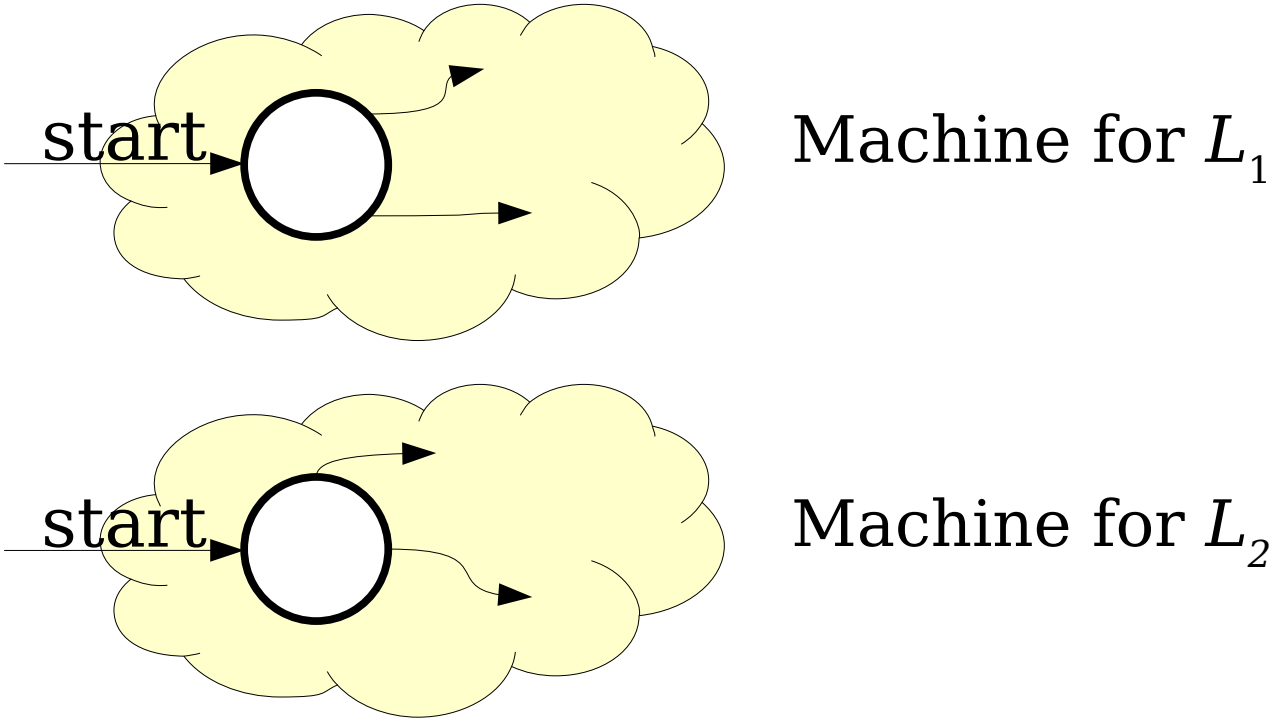

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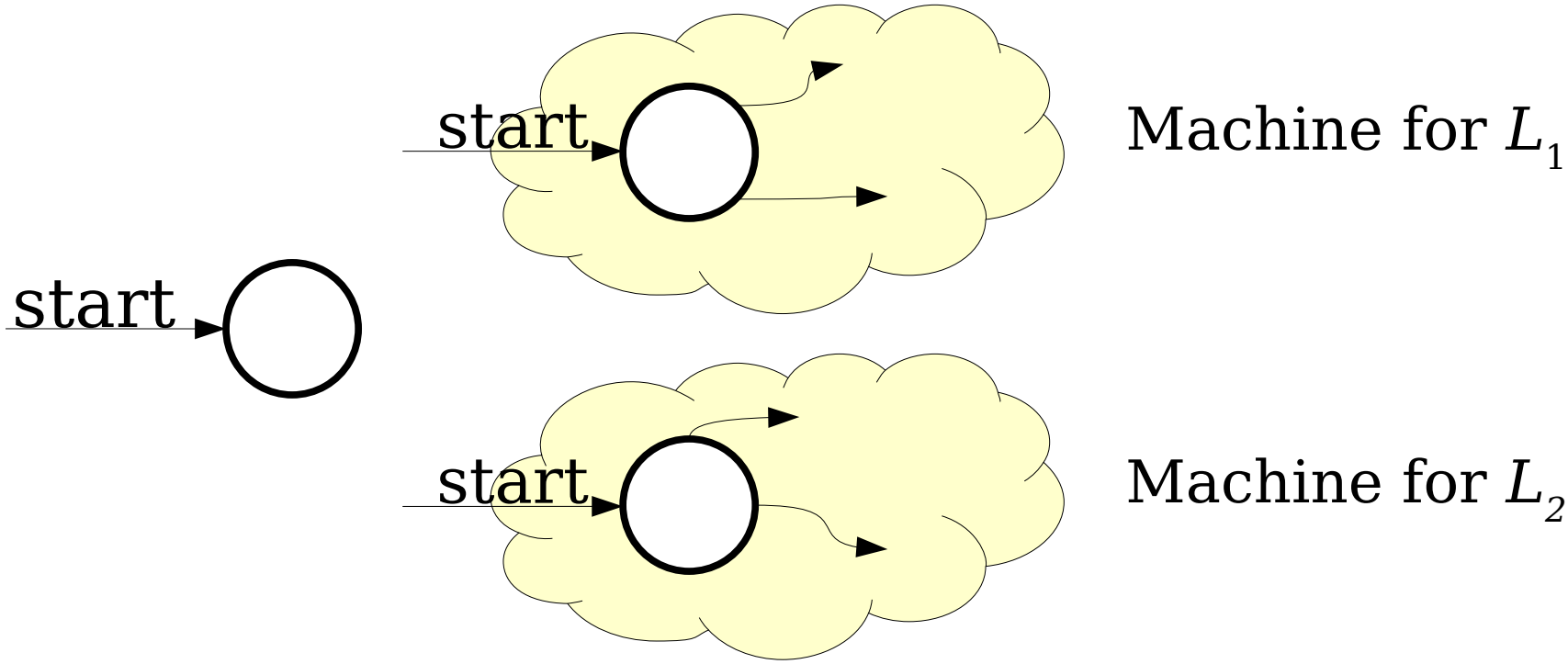

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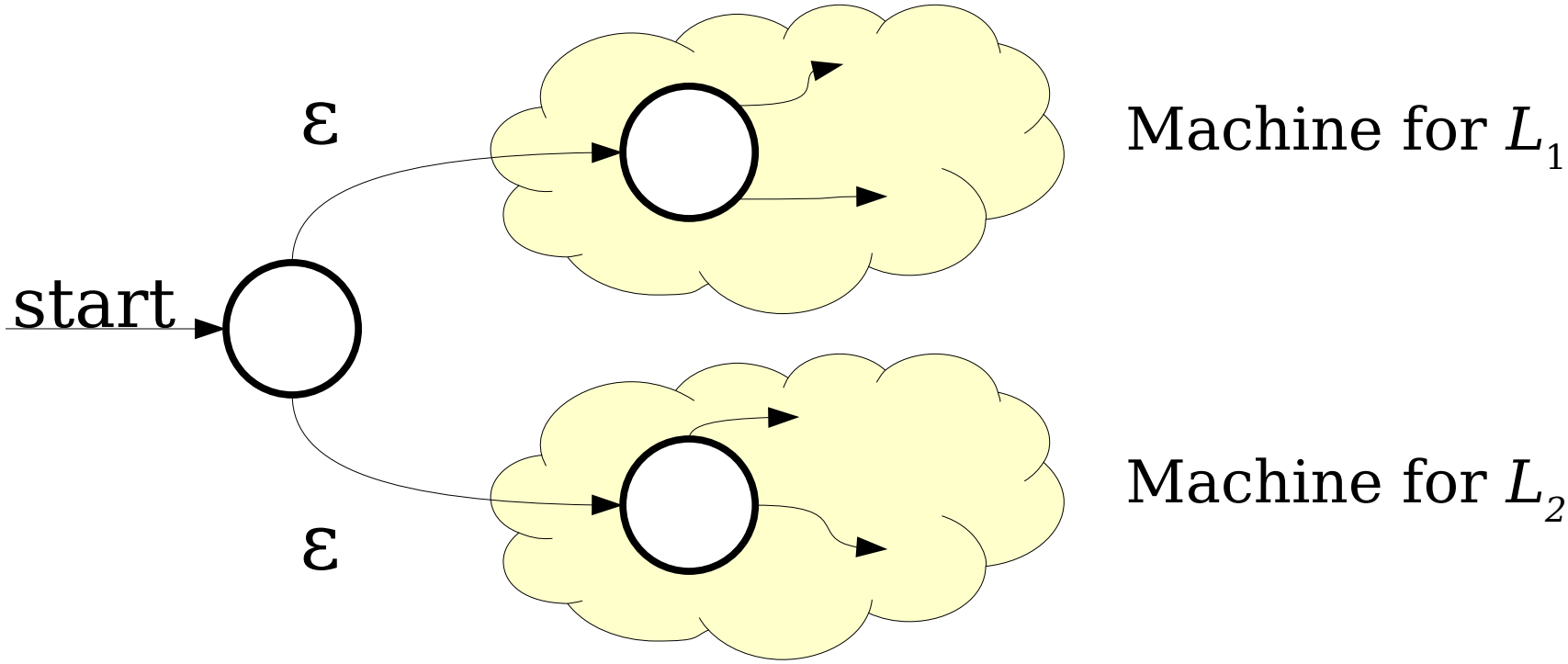

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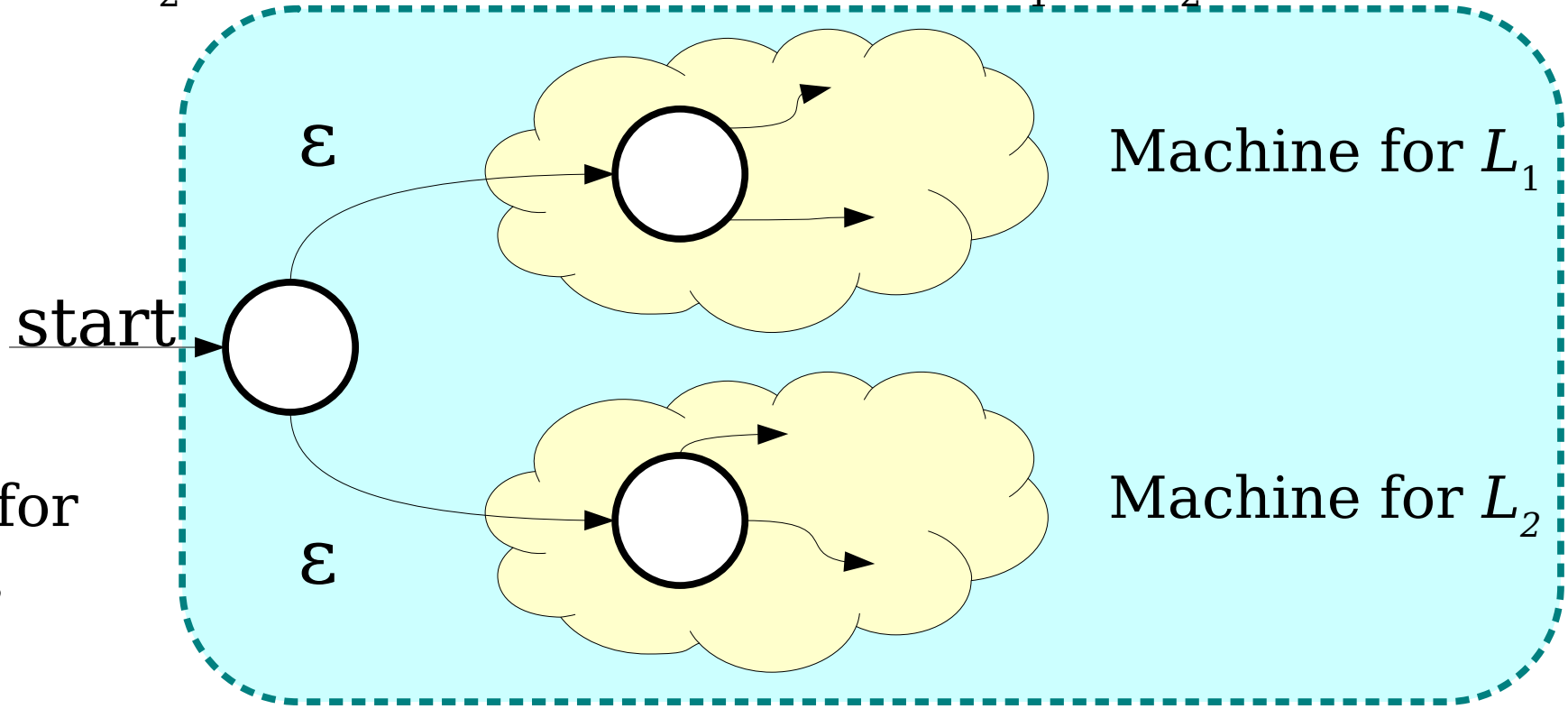

Machine for $L_1 \cup L_2$

The Intersection of Two Languages

- If L_1 and L_2 are languages over Σ, then $L_1 \cap L_2$ is the language of strings in both L_1 and L_2.
- Question: If L_1 and L_2 are regular, is $L_1 \cap L_2$ regular as well?

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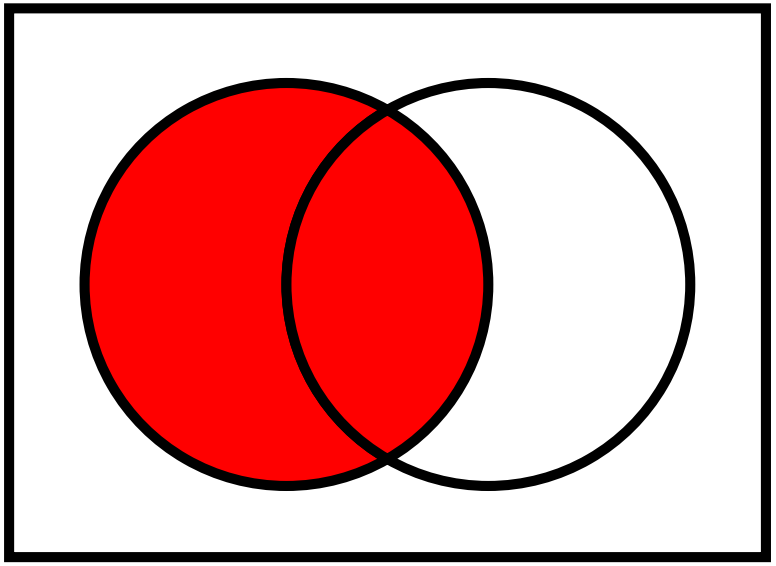
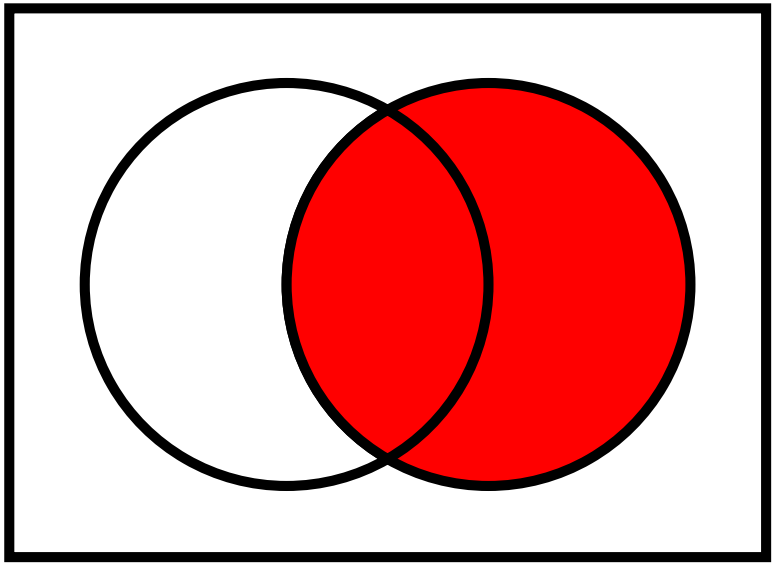


L_1



L_2

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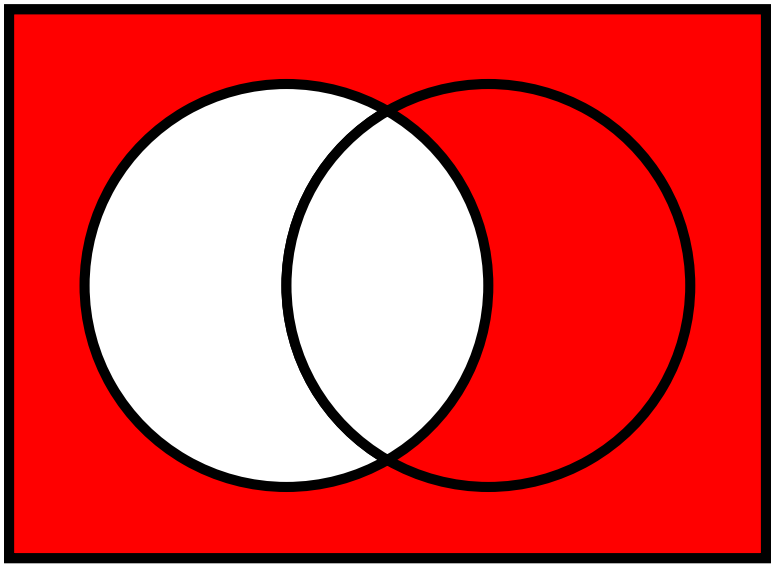
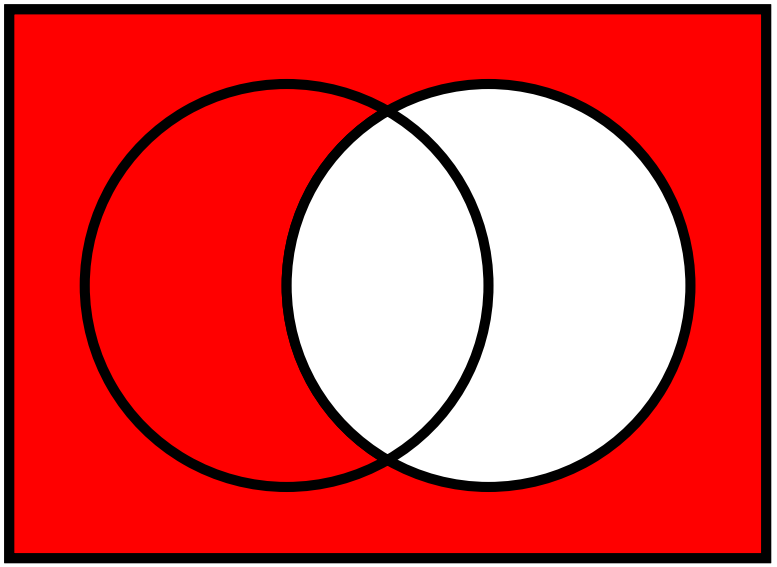


$\overline{L}_1$



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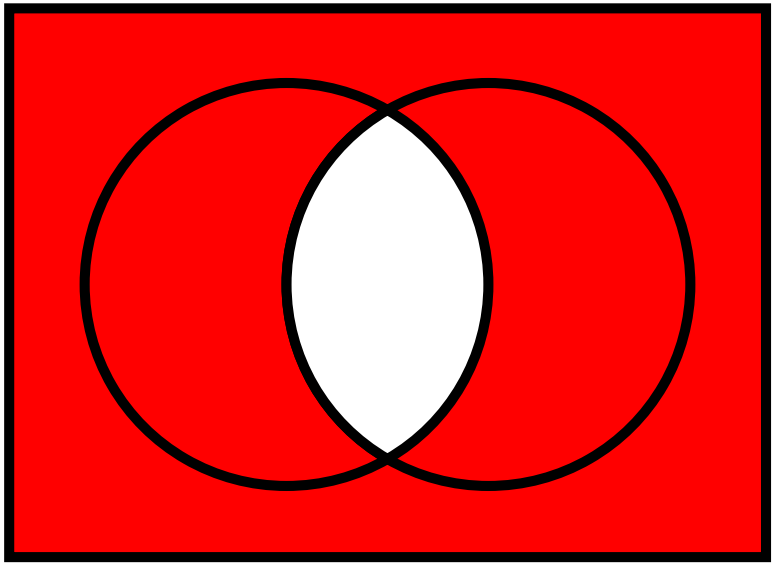

$$\overline{L}_1 \cup \overline{L}_2$$

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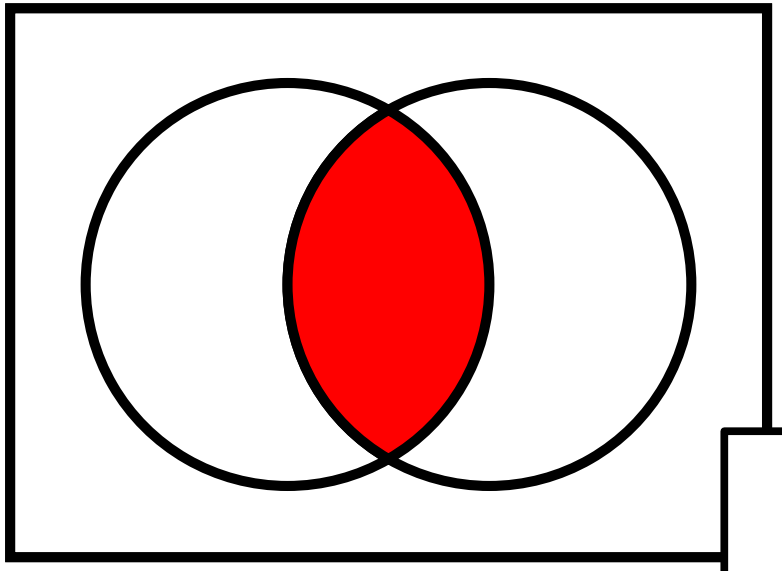

$$\overline{\overline{L_1} \cup \overline{L_2}}$$

Hey, it's De Morgan's laws!

Concatenation

String Concatenation

- If $w \in \Sigma^*$ and $x \in \Sigma^*$, the ***concatenation*** of w and x, denoted ***wx***, is the string formed by tacking all the characters of x onto the end of w.
- Example: if w = `quo` and x = `kka`, the concatenation wx = `quokka`.
- This is analogous to the + operator for strings in many programming languages.
- Some facts about concatenation:
  - The empty string ε is the ***identity element*** for concatenation:

$$w\varepsilon = \varepsilon w = w$$

  - Concatenation is ***associative***:

$$wxy = w(xy) = (wx)y$$

Concatenation

- The ***concatenation*** of two languages L_1 and L_2 over the alphabet Σ is the language

$$L_1L_2 = \{ wx \in \Sigma^* \mid w \in L_1 \wedge x \in L_2 \}$$

Concatenation Example

- Let Σ = { `a`, `b`, …, `z`, `A`, `B`, …, `Z` } and consider these languages over Σ:
  - ***Noun*** = { `Puppy`, `Rainbow`, `Whale`, … }
  - ***Verb*** = { `Hugs`, `Juggles`, `Loves`, … }
  - ***The*** = { `The` }
- The language ***TheNounVerbTheNoun*** is
  - { `ThePuppyHugsTheWhale`, `TheWhaleLovesTheRainbow`, `TheRainbowJugglesTheRainbow`, … }

Concatenation

- The ***concatenation*** of two languages L_1 and L_2 over the alphabet Σ is the language

$$L_1L_2 = \{ wx \in \Sigma^* \mid w \in L_1 \wedge x \in L_2 \}$$

- Two views of L_1L_2:
  - The set of all strings that can be made by concatenating a string in L_1 with a string in L_2.
  - The set of strings that can be split into two pieces: a piece from L_1 and a piece from L_2.

Concatenating Regular Languages

- If L_1 and L_2 are regular languages, is L_1L_2?
- Intuition – can we split a string w into two strings xy such that $x \in L_1$ and $y \in L_2$?

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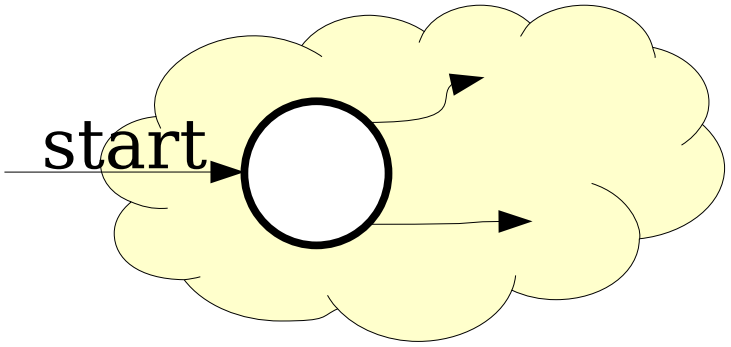
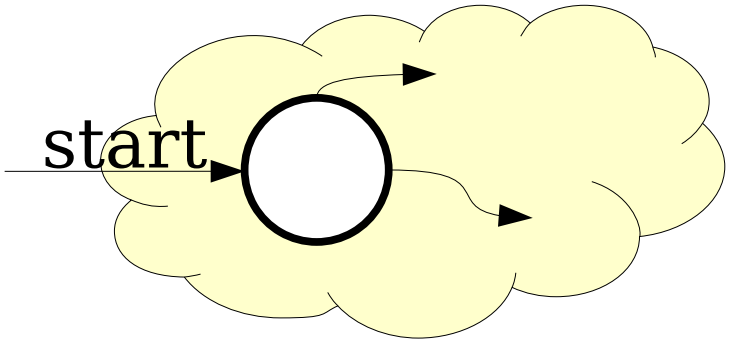


Machine for L_1



Machine for L_2

Concatenating Regular Languages

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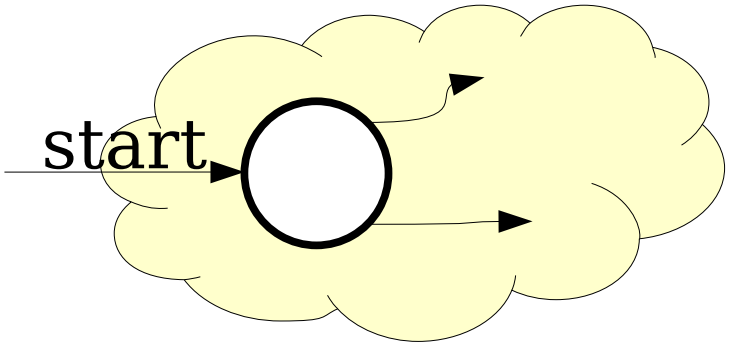

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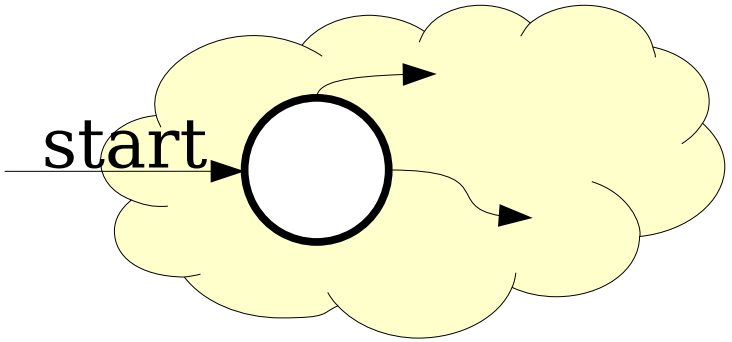

Machine for L_2

b	**o**	**o**	**k**	**k**	**e**	**e**	**p**	**e**	**r**

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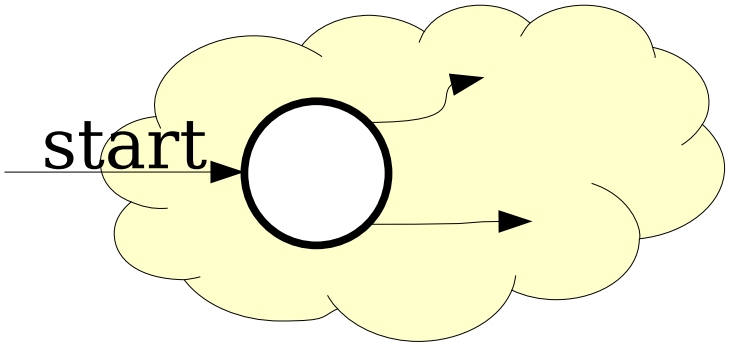

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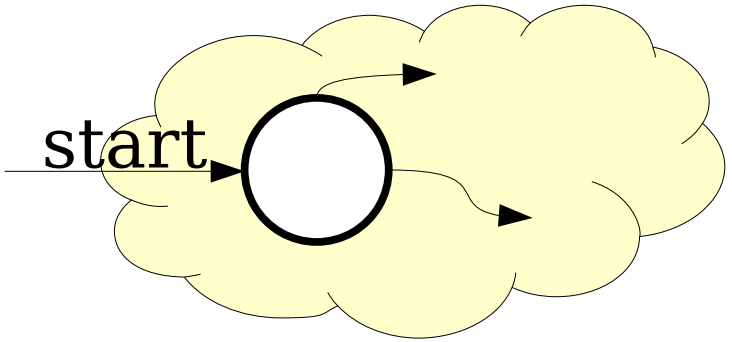

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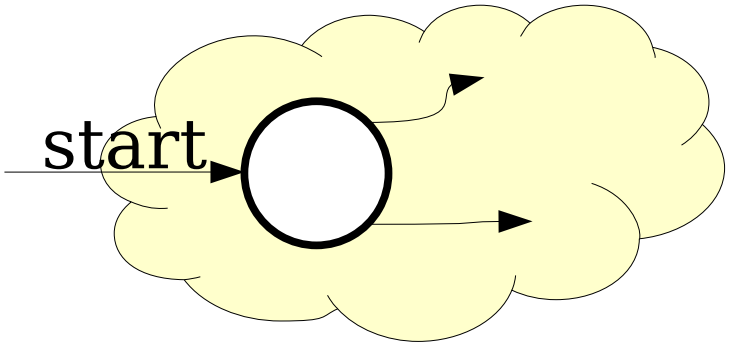

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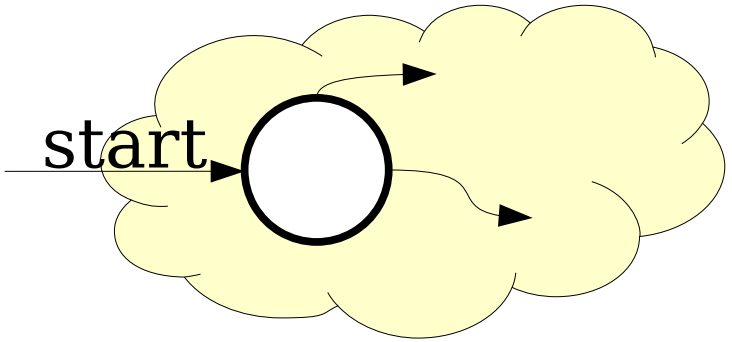


Machine for L_2

b	o	o	k

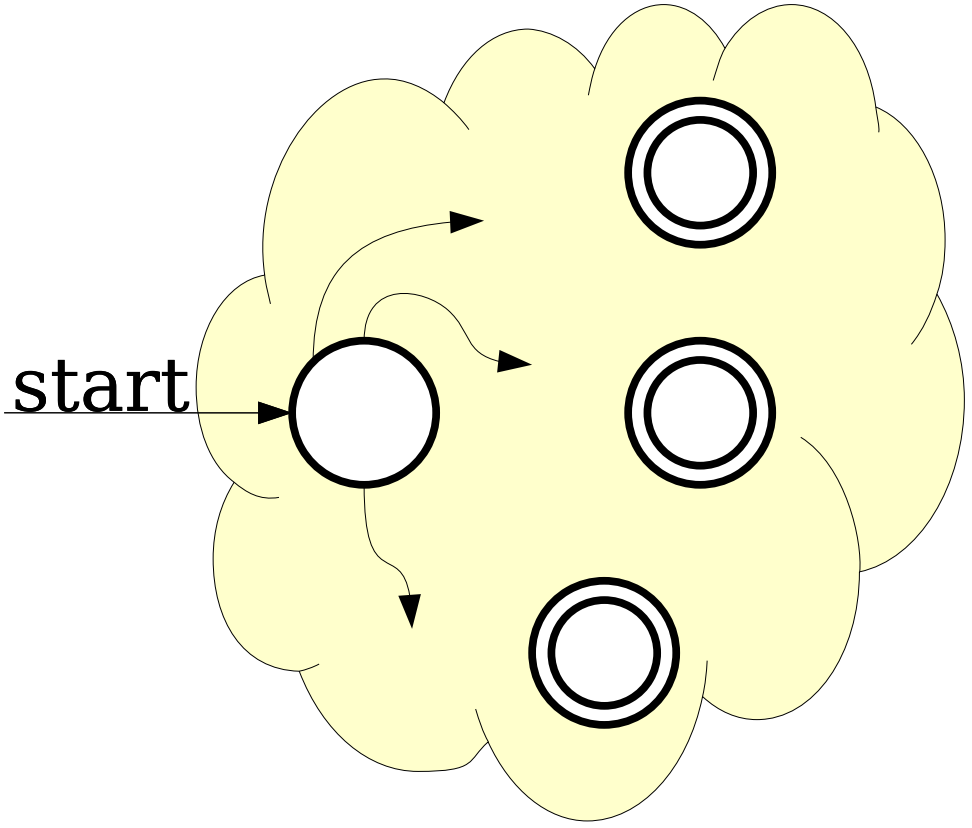
k	e	e	p	e	r

Concatenating Regular Languages

- If L_1 and L_2 are regular languages, is L_1L_2?
- Intuition – can we split a string w into two strings xy such that $x \in L_1$ and $y \in L_2$?
- ***Idea***:
  - Run a DFA/NFA for L_1 on w.
  - Whenever it reaches an accepting state, optionally hand the rest of w to a DFA/NFA for L_2.
  - If the automaton for L_2 accepts the rest, $w \in L_1L_2$.
  - If the automaton for L_2 rejects the remainder, the split was incorrect.

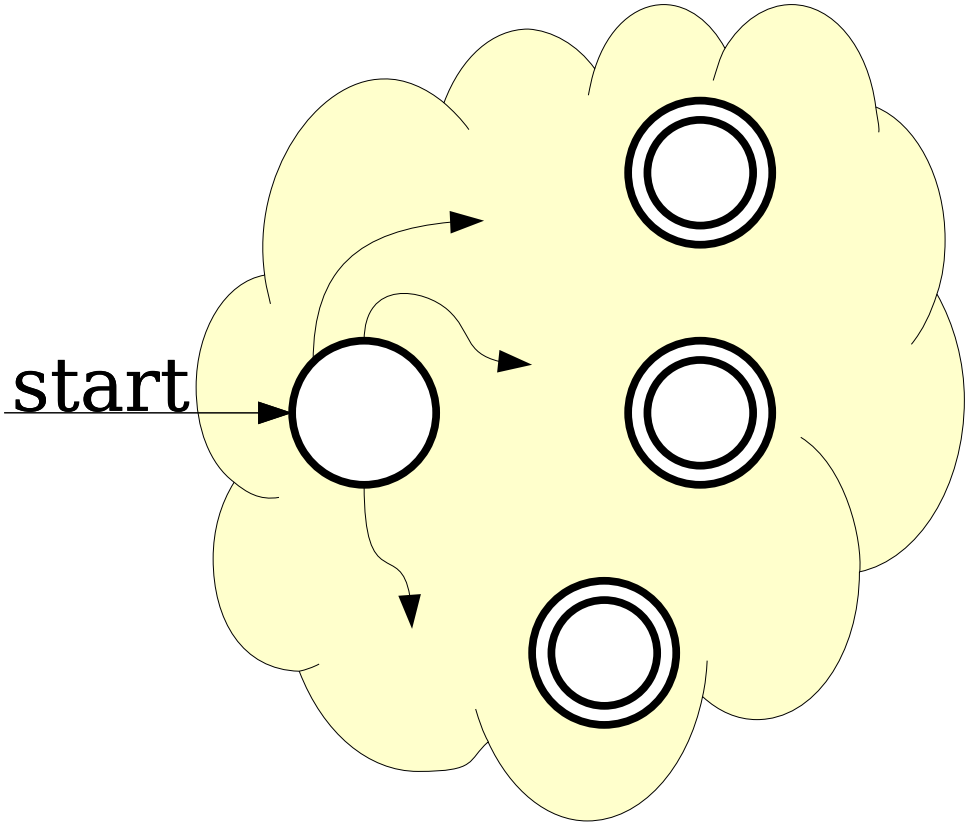
Concatenating Regular Languages

Concatenating Regular Languages



Machine for L_1

Concatenating Regular Languages



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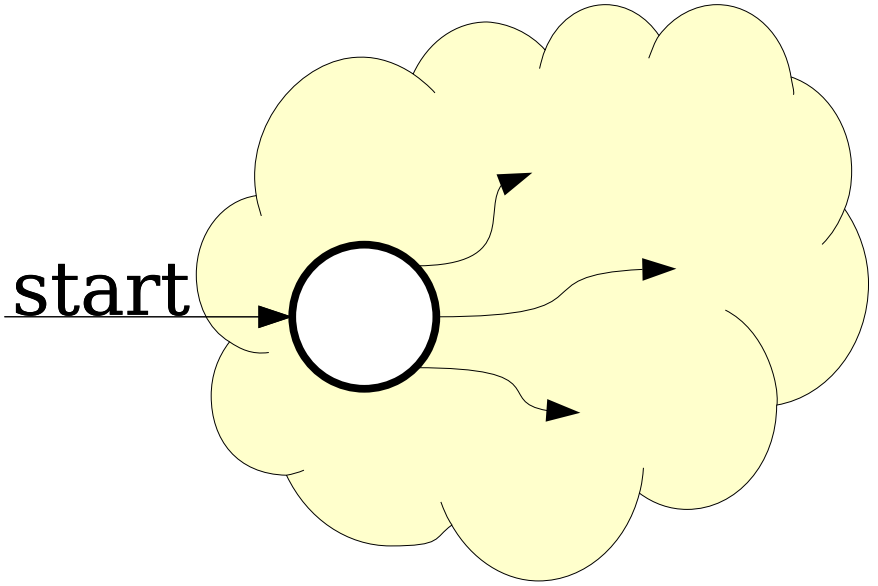
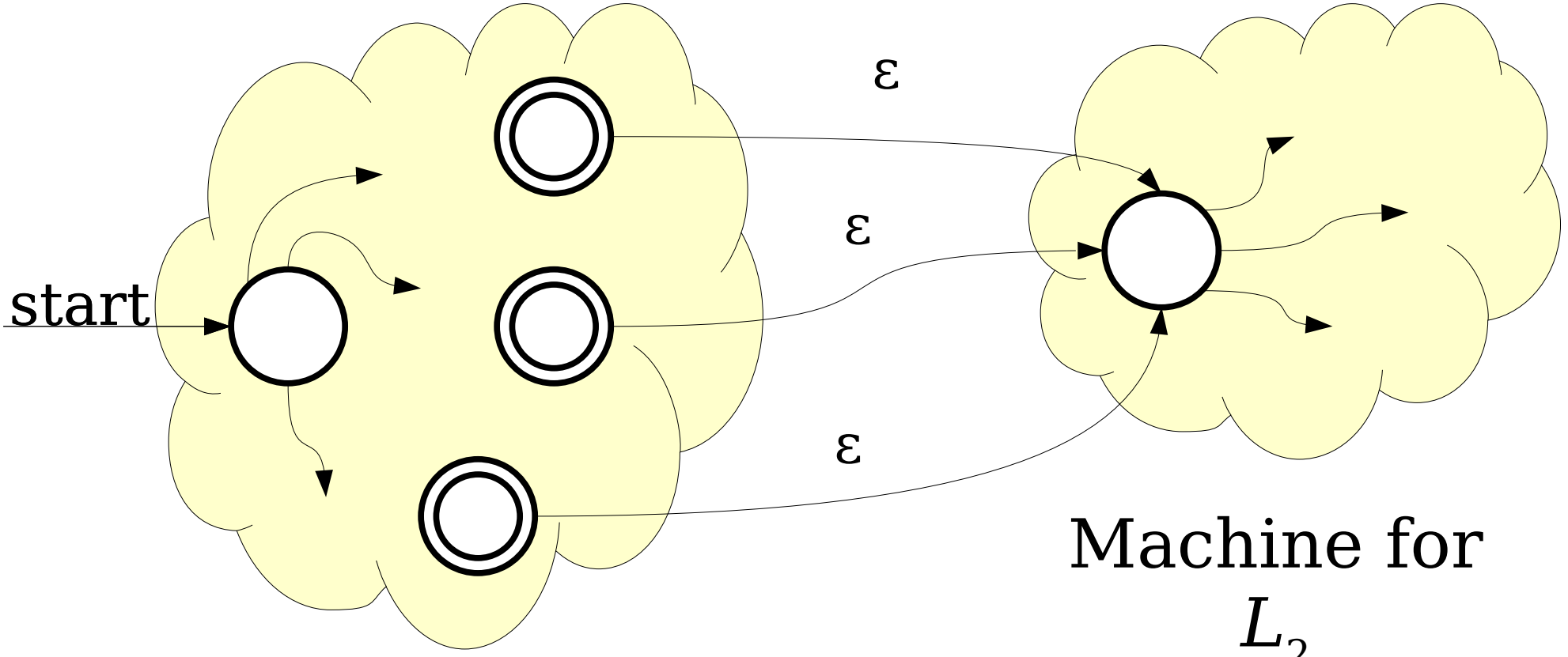


Machine for
L_2

Concatenating Regular Languages



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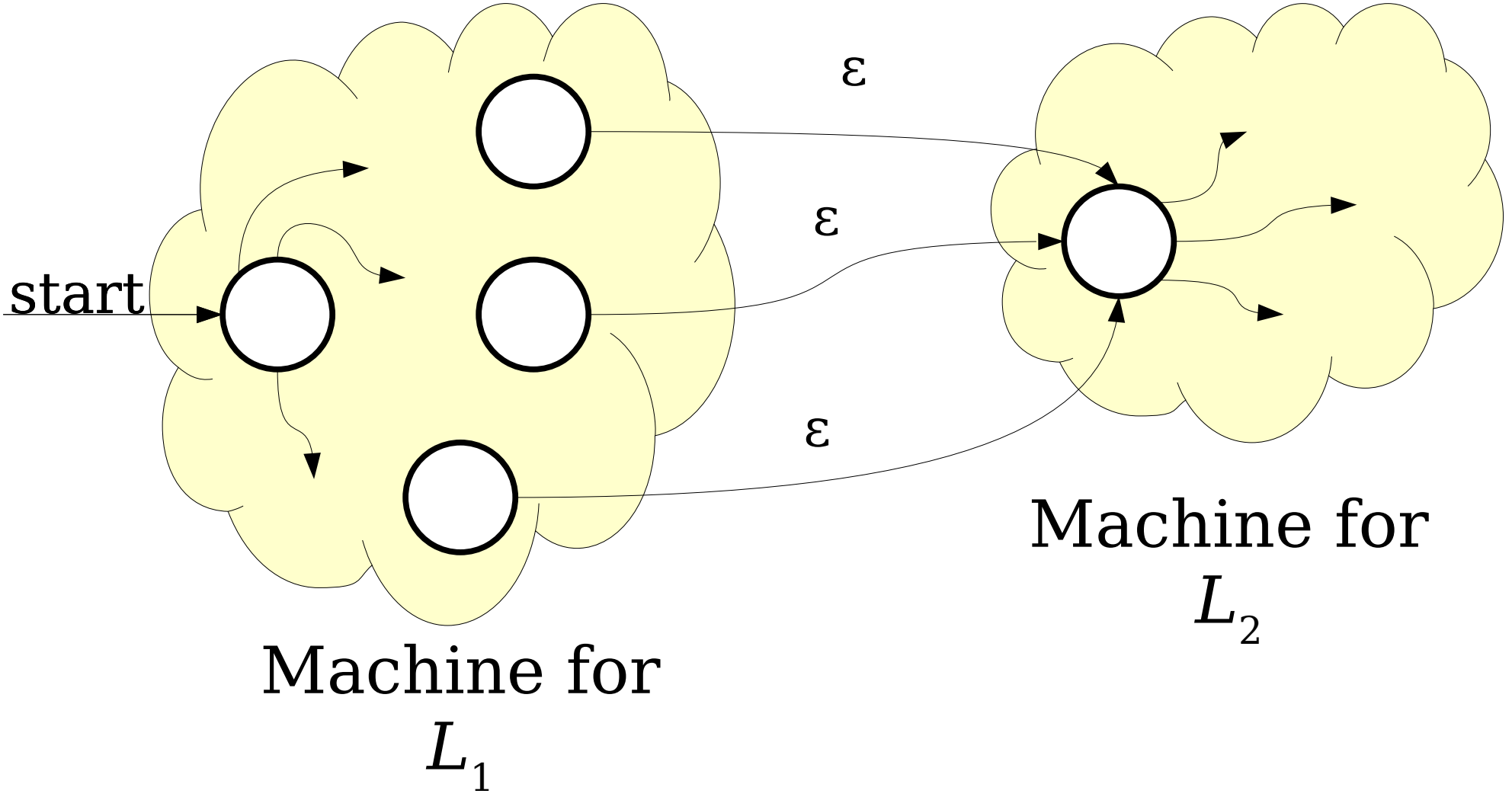

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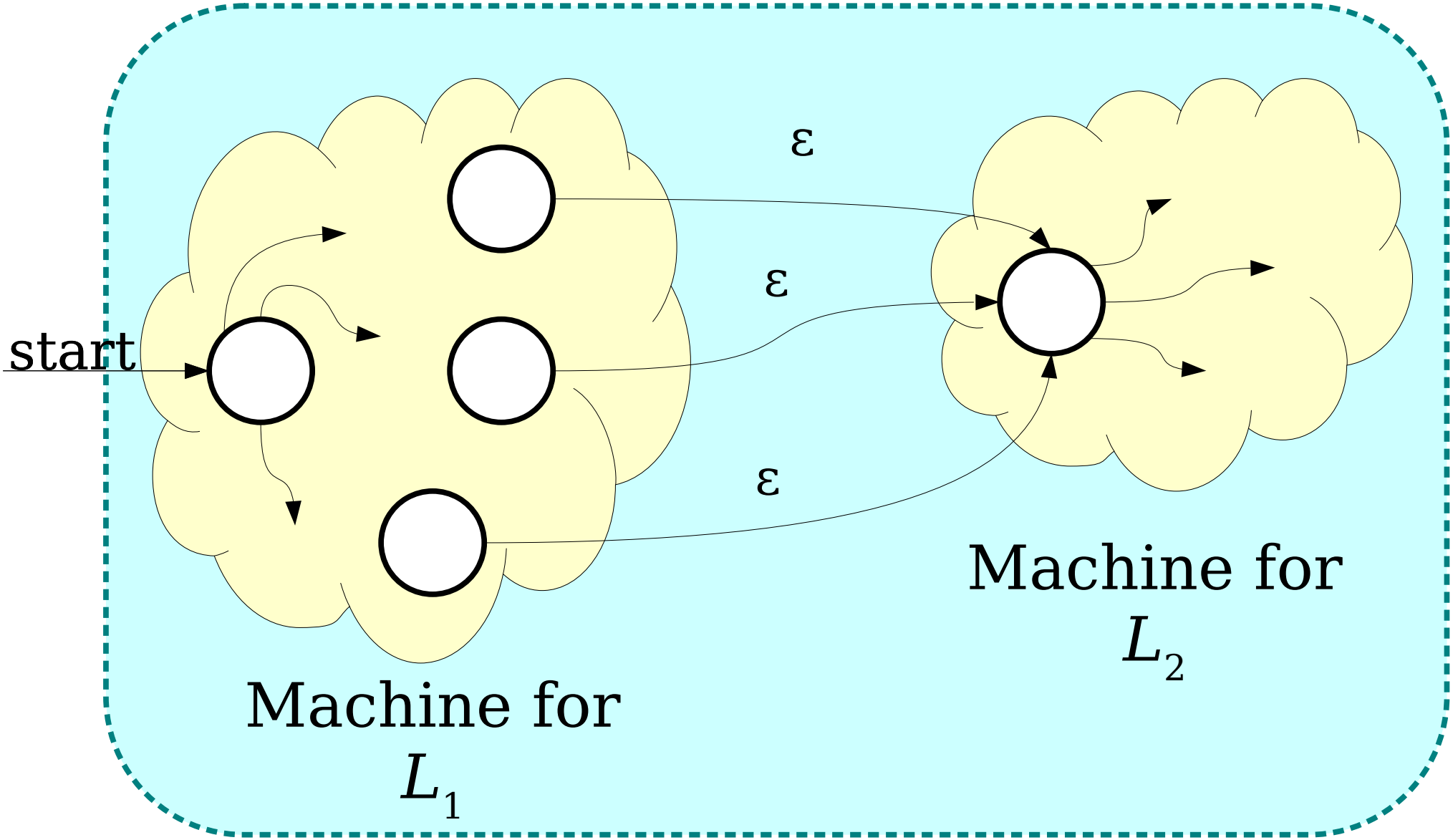


Machine for $L_1 L_2$

Lots and Lots of Concatenation

- Consider the language L = { **aa**, **b** }
- LL is the set of strings formed by concatenating pairs of strings in L.

{ **aaaa**, **aab**, **baa**, **bb** }

- LLL is the set of strings formed by concatenating triples of strings in L.

{ **aaaaaa**, **aaaab**, **aabaa**, **aabb**, **baaaa**, **baab**, **bbaa**, **bbb**}

- $LLLL$ is the set of strings formed by concatenating quadruples of strings in L.

{ **aaaaaaaa**, **aaaaaab**, **aaaabaa**, **aaaabb**, **aabaaaa**,
aabaab, **aabbaa**, **aabbb**, **baaaaaa**, **baaaab**, **baabaa**,
baabb, **bbaaaa**, **bbaab**, **bbbaa**, **bbbb**}

Language Exponentiation

- We can define what it means to “exponentiate” a language as follows:
- $L^0 = \{\varepsilon\}$
  - Intuition: The only string you can form by gluing no strings together is the empty string.
  - Notice that $\{\varepsilon\} \neq \varnothing$. Can you explain why?
- $L^{n+1} = LL^n$
  - Idea: Concatenating (n+1) strings together works by concatenating n strings, then concatenating one more.
- ***Question to ponder:*** Why define $L^0 = \{\varepsilon\}$?
- ***Question to ponder:*** What is $\varnothing^0$?

The Kleene Star

The Kleene Closure

- An important operation on languages is the ***Kleene Closure***, which is defined as

$$L^* = \{ w \in \Sigma^* \mid \exists n \in \mathbb{N}.\ w \in L^n \}$$

- Mathematically:

$$w \in L^* \quad \leftrightarrow \quad \exists n \in \mathbb{N}.\ w \in L^n$$

- Intuitively, L^* is the language all possible ways of concatenating zero or more strings in L together, possibly with repetition.
- ***Question to ponder:*** What is Ø*?

The Kleene Closure

If L = { **a**, **bb** }, then L^* = {

ε,

a, **bb**,

aa, **abb**, **bba**, **bbbb**,

aaa, **aabb**, **abba**, **abbbb**, **bbaa**, **bbabb**, **bbbba**, **bbbbbb**,

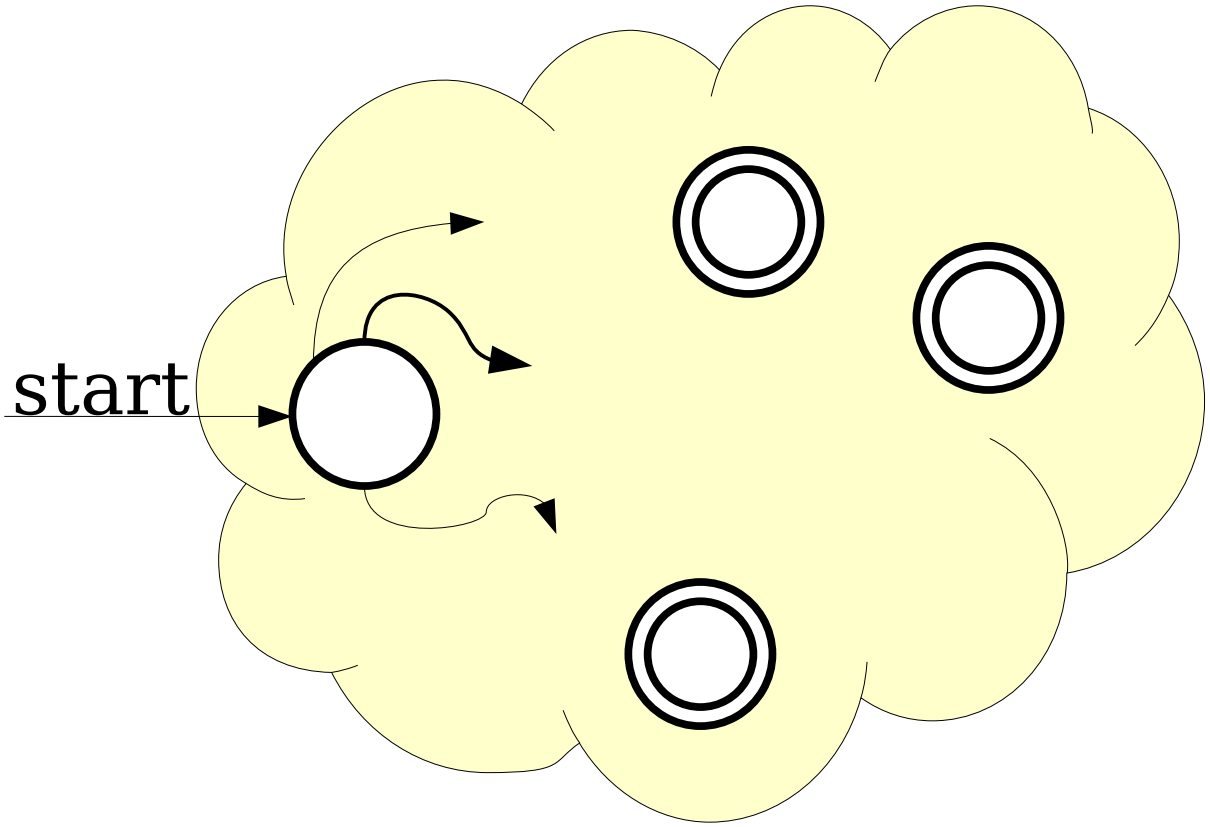
…

}

Think of L^* as the set of strings you can make if you have a collection of stamps – one for each string in L – and you form every possible string that can be made from those stamps.

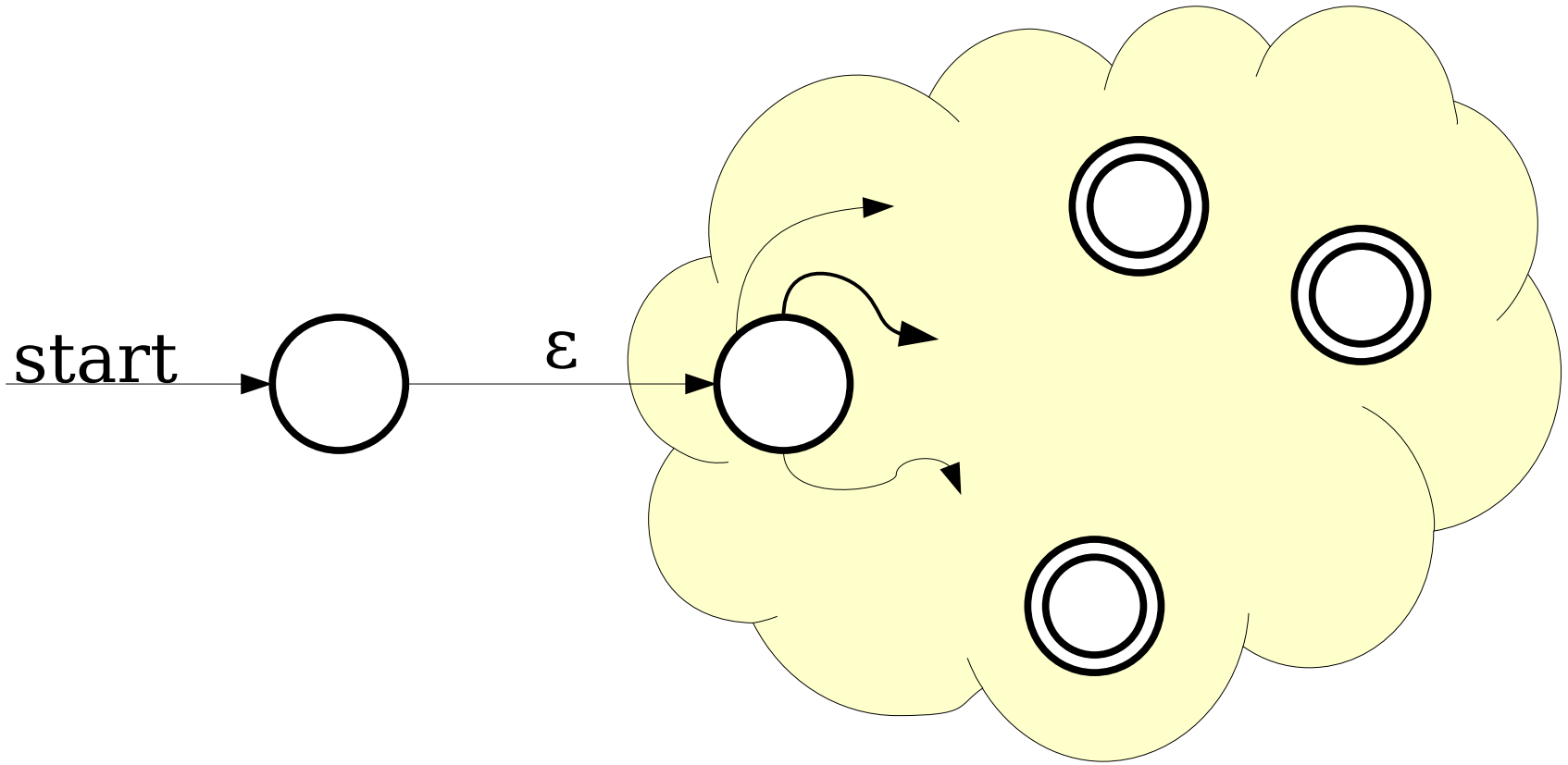
Idea: Can we convert an NFA for language L to an NFA for language L*?

The Kleene Star



Machine for L

The Kleene Star



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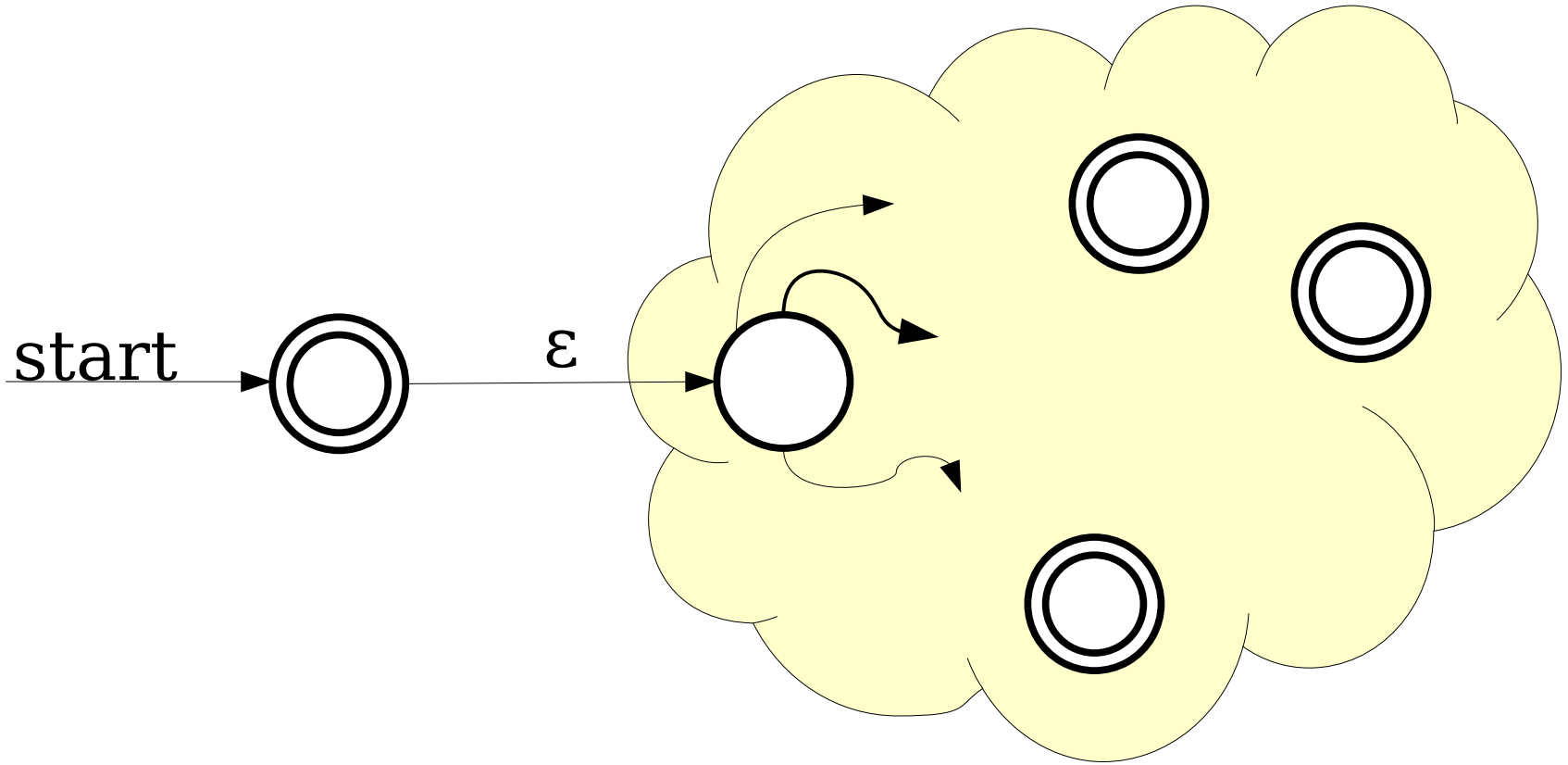

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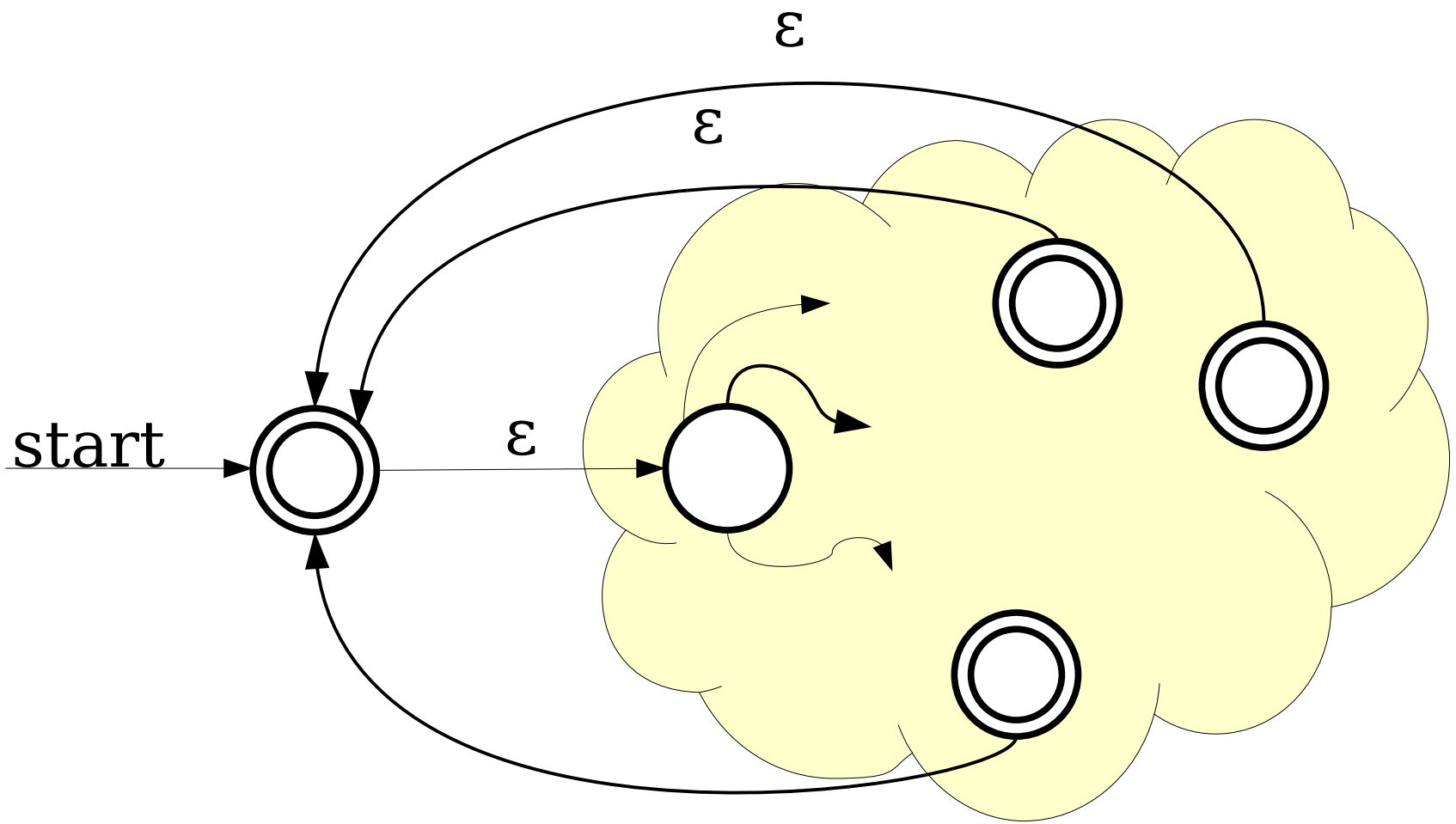

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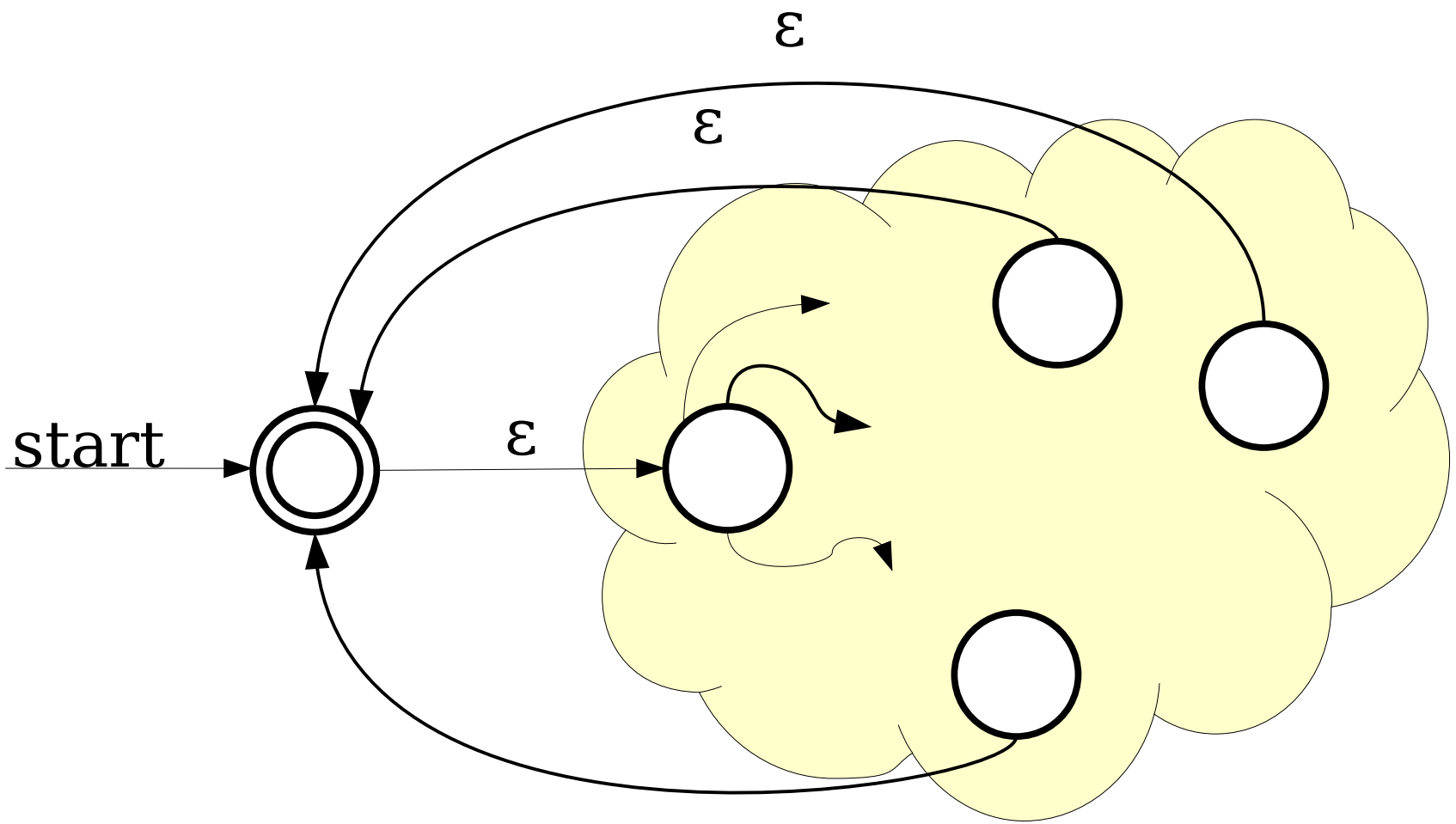

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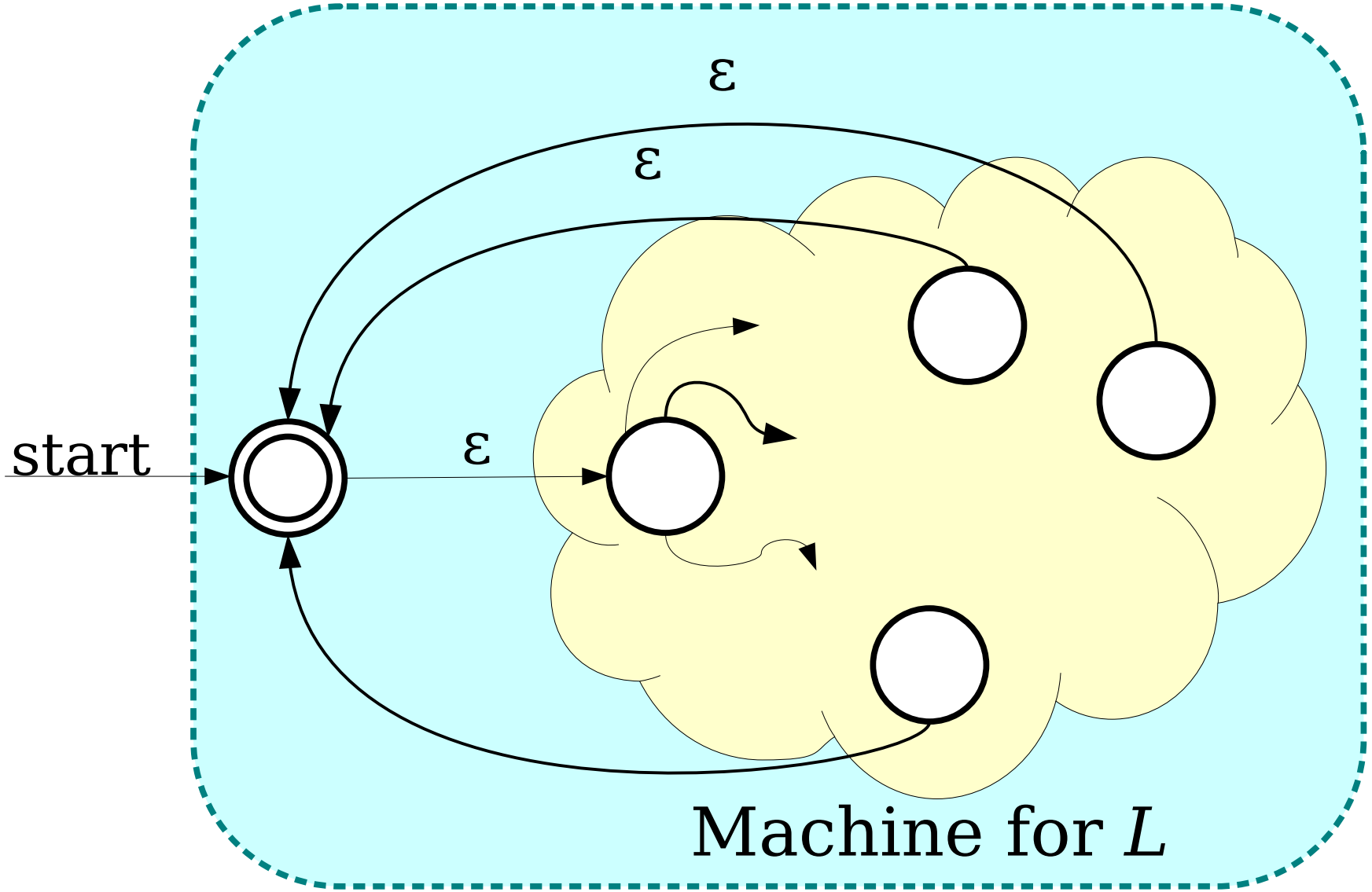
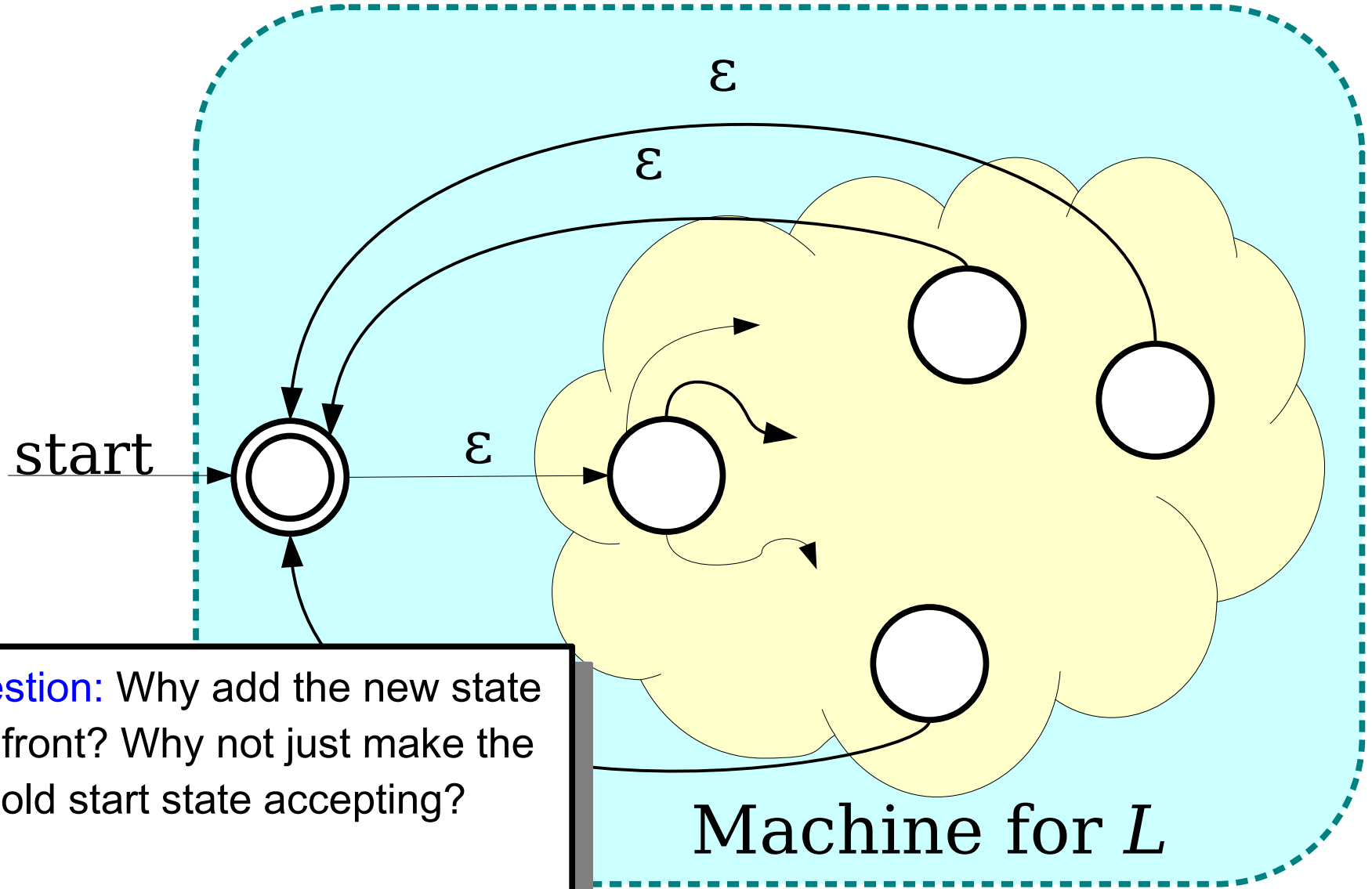


Machine for L^*

The Kleene Star



Question: Why add the new state out front? Why not just make the old start state accepting?

achine for L^*

Closure Properties

- ***Theorem:*** If L_1 and L_2 are regular languages over an alphabet Σ, then so are the following languages:
  - $\overline{L}_1$
  - $L_1 \cup L_2$
  - $L_1 \cap L_2$
  - $L_1 L_2$
  - L_1^*
- These properties are called ***closure properties of the regular languages***.

Next Time

- ***Regular Expressions***
  - Building languages from the ground up!
- ***Thompson's Algorithm***
  - A UNIX Programmer in Theoryland.
- ***Kleene's Theorem***
  - From machines to programs!

Regular Expressions

Recap from Last Time

Regular Languages

- A language L is a ***regular language*** if there is a DFA D such that $\mathscr{L}(D) = L$.
- ***Theorem:*** The following are equivalent:
  - L is a regular language.
  - There is a DFA for L.
  - There is an NFA for L.

Language Concatenation

- If $w \in \Sigma^*$ and $x \in \Sigma^*$, then wx is the ***concatenation*** of w and x.
- If L_1 and L_2 are languages over Σ, the ***concatenation*** of L_1 and L_2 is the language L_1L_2 defined as

$$\boldsymbol{L_1L_2 = \{\, wx \mid w \in L_1 \text{ and } x \in L_2 \,\}}$$

- Example: if $L_1 = \{$ **a**, **ba**, **bb** $\}$ and $L_2 = \{$ **aa**, **bb** $\}$, then

$$L_1L_2 = \{ \textbf{aaa}, \textbf{abb}, \textbf{baaa}, \textbf{babb}, \textbf{bbaa}, \textbf{bbbb} \}$$

Lots and Lots of Concatenation

- Consider the language L = { **aa**, **b** }
- LL is the set of strings formed by concatenating pairs of strings in L.

{ **aaaa**, **aab**, **baa**, **bb** }

- LLL is the set of strings formed by concatenating triples of strings in L.

{ **aaaaaa**, **aaaab**, **aabaa**, **aabb**, **baaaa**, **baab**, **bbaa**, **bbb**}

- $LLLL$ is the set of strings formed by concatenating quadruples of strings in L.

{ **aaaaaaaa**, **aaaaaab**, **aaaabaa**, **aaaabb**, **aabaaaa**,
aabaab, **aabbaa**, **aabbb**, **baaaaaa**, **baaaab**, **baabaa**,
baabb, **bbaaaa**, **bbaab**, **bbbaa**, **bbbb**}

Language Exponentiation

- We can define what it means to “exponentiate” a language as follows:
- $L^0 = \{\varepsilon\}$
  - Intuition: The only string you can form by gluing no strings together is the empty string.
  - Notice that $\{\varepsilon\} \neq \varnothing$. Can you explain why?
- $L^{n+1} = LL^n$
  - Idea: Concatenating (n+1) strings together works by concatenating n strings, then concatenating one more.
- ***Question to ponder:*** Why define $L^0 = \{\varepsilon\}$?
- ***Question to ponder:*** What is $\varnothing^0$?

The Kleene Closure

- An important operation on languages is the ***Kleene Closure***, which is defined as

$$L^* = \{ w \in \Sigma^* \mid \exists n \in \mathbb{N}.\ w \in L^n \}$$

- Mathematically:

$$w \in L^* \quad \textbf{iff} \quad \exists n \in \mathbb{N}.\ w \in L^n$$

- Intuitively, all possible ways of concatenating zero or more strings in L together, possibly with repetition.
- ***Question:*** What is $\emptyset^0$?

The Kleene Closure

If L = { **a**, **bb** }, then L^* = {

ε,

a, **bb**,

aa, **abb**, **bba**, **bbbb**,

aaa, **aabb**, **abba**, **abbbb**, **bbaa**, **bbabb**, **bbbba**, **bbbbbb**,

...

}

Think of L^* as the set of strings you can make if you have a collection of stamps – one for each string in L – and you form every possible string that can be made from those stamps.

Closure Properties

- ***Theorem:*** If L_1 and L_2 are regular languages over an alphabet Σ, then so are the following languages:
  - $\overline{L}_1$
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  - $L_1 \cap L_2$
  - $L_1 L_2$
  - L_1^*
- These properties are called ***closure properties of the regular languages***.

New Stuff!

Another View of Regular Languages

Rethinking Regular Languages

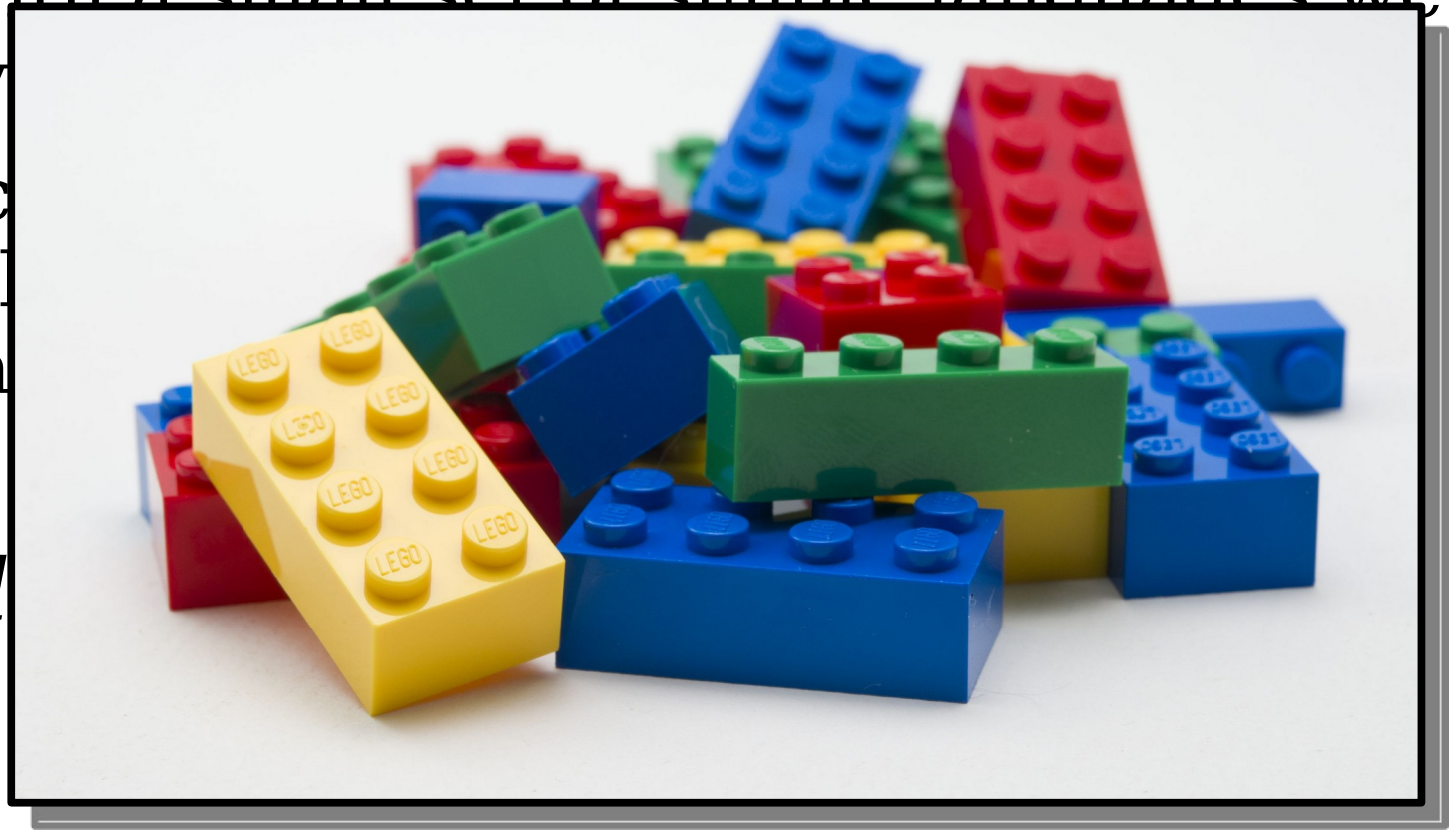
- We currently have several tools for showing a language L is regular:
  - Construct a DFA for L.
  - Construct an NFA for L.
  - Combine several simpler regular languages together via closure properties to form L.
- We have not spoken much of this last idea.

Constructing Regular Languages

- ***Idea:*** Build up all regular languages as follows:
  - Start with a small set of simple languages we already know to be regular.
  - Using closure properties, combine these simple languages together to form more elaborate languages.
- *This is a bottom-up approach to the regular languages.*

Constructing Regular Languages

- ***Idea:*** Build up all regular languages as follows:
  - Start with a small set of simple languages we already
  - Using c
    simple l
    elabora
- *This is a*
  regular l



Regular Expressions

- ***Regular expressions*** are a way of describing a language via a string representation.
- They're used just about everywhere:
  - They're built into the JavaScript language and used for data validation.
  - They're used in the UNIX `grep` and `flex` tools to search files and build compilers.
  - They're employed to clean and scrape data for large-scale analysis projects.
- Conceptually, regular expressions are strings describing how to assemble a larger language out of smaller pieces.

Atomic Regular Expressions

- The regular expressions begin with three simple building blocks.
- The symbol **Ø** is a regular expression that represents the empty language Ø.
- For any **a** ∈ Σ, the symbol **a** is a regular expression for the language {**a**}.
- The symbol **ε** is a regular expression that represents the language {ε}.
  - ***Remember:* {ε} ≠ Ø!**
  - ***Remember:* {ε} ≠ ε!**

Compound Regular Expressions

- If R_1 and R_2 are regular expressions, $\boldsymbol{R_1 R_2}$ is a regular expression for the *concatenation* of the languages of R_1 and R_2.
- If R_1 and R_2 are regular expressions, $\boldsymbol{R_1 \cup R_2}$ is a regular expression for the *union* of the languages of R_1 and R_2.
- If R is a regular expression, $\boldsymbol{R^*}$ is a regular expression for the *Kleene closure* of the language of R.
- If R is a regular expression, $\boldsymbol{(R)}$ is a regular expression with the same meaning as R.

Operator Precedence

- Here's the operator precedence for regular expressions:

$$(R)$$

$$R*$$

$$R_1 R_2$$

$$R_1 \cup R_2$$

- So `ab*cUd` is parsed as `((a(b*))c)Ud`

Regular Expressions, Formally

- The ***language of a regular expression*** is the language described by that regular expression.
- Formally:
  - $\mathcal{L}(\boldsymbol{\varepsilon}) = \{\varepsilon\}$
  - $\mathcal{L}(\boldsymbol{\emptyset}) = \emptyset$
  - $\mathcal{L}(\mathbf{a}) = \{\mathbf{a}\}$
  - $\mathcal{L}(R_1 R_2) = \mathcal{L}(R_1)\ \mathcal{L}(R_2)$
  - $\mathcal{L}(R_1 \cup R_2) = \mathcal{L}(R_1) \cup \mathcal{L}(R_2)$
  - $\mathcal{L}(R^*) = \mathcal{L}(R)^*$
  - $\mathcal{L}(\mathbf{(}R\mathbf{)}) = \mathcal{L}(R)$

Worthwhile activity: Apply this recursive definition to

a(b∪c)((d))

and see what you get.

Designing Regular Expressions

- Let $\Sigma = \{$`a`, `b`$\}$.
- Let $L = \{ w \in \Sigma^* \mid w$ contains `aa` as a substring $\}$.

Designing Regular Expressions

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(a ∪ b)*aa(a ∪ b)*

Designing Regular Expressions

- Let $\Sigma = \{$`a`, `b`$\}$.
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`(a U b)*aa(a U b)*`

Designing Regular Expressions

- Let $\Sigma = \{$**a**, **b**$\}$.
- Let $L = \{ w \in \Sigma^* \mid w$ contains **aa** as a substring $\}$.

(a ∪ b)*aa(a ∪ b)*

bbabbbaabab
aaaa
bbbbbabbbbaabbbbb

Designing Regular Expressions

- Let $\Sigma = \{$`a`, `b`$\}$.
- Let $L = \{ w \in \Sigma^* \mid w$ contains `aa` as a substring $\}$.

`(a ∪ b)*aa(a ∪ b)*`

`bbabbbaabab`
`aaaa`
`bbbbbabbbbaabbbbb`

Designing Regular Expressions

- Let $\Sigma = \{$**a**, **b**$\}$.
- Let $L = \{ w \in \Sigma^* \mid w$ contains **aa** as a substring $\}$.

Σ*aaΣ*

bbabbbaabab

aaaa

bbbbbabbbbaabbbbb

Designing Regular Expressions

- Let $\Sigma = \{$`a`, `b`$\}$.
- Let $L = \{ w \in \Sigma^* \mid |w| = 4 \}$.

Designing Regular Expressions

Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

Let $L = \{ w \in \Sigma^* \mid |w| = 4 \}$.

The length of a string w is denoted $|w|$

Designing Regular Expressions

- Let $\Sigma = \{$`a`, `b`$\}$.
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Designing Regular Expressions

- Let $\Sigma = \{$**a**, **b**$\}$.
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ΣΣΣΣ

Designing Regular Expressions

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Designing Regular Expressions

- Let $\Sigma = \{\texttt{a}, \texttt{b}\}$.
- Let $L = \{ w \in \Sigma^* \mid |w| = 4 \}$.

$$\Sigma\Sigma\Sigma\Sigma$$

aaaa
baba
bbbb
baaa

Designing Regular Expressions

- Let $\Sigma = \{\texttt{a}, \texttt{b}\}$.
- Let $L = \{ w \in \Sigma^* \mid |w| = 4 \}$.

ΣΣΣΣ

aaaa
baba
bbbb
baaa

Designing Regular Expressions

- Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.
- Let $L = \{ w \in \Sigma^* \mid |w| = 4 \}$.

$$\Sigma^4$$

aaaa
baba
bbbb
baaa

Designing Regular Expressions

- Let $\Sigma = \{$**a**, **b**$\}$.
- Let $L = \{ w \in \Sigma^* \mid |w| = 4 \}$.

$$\Sigma^4$$

aaaa
baba
bbbb
baaa

Designing Regular Expressions

- Let $\Sigma = \{$`a`, `b`$\}$.
- Let $L = \{ w \in \Sigma^* \mid w$ contains at most one `a` $\}$.

Which of the following is a regular expression for L?

A) `Σ*aΣ*`
B) `b*ab* U b*`
C) `b*(a U ε)b*`
D) `b*a*b* U b*`
E) `b*(a* U ε)b*`
F) None of the above, or two or more of the above.

Respond at pollev.com/zhenglian

Designing Regular Expressions

- Let $\Sigma = \{$**a**, **b**$\}$.
- Let $L = \{ w \in \Sigma^* \mid w$ contains at most one **a** $\}$.

b*(a ∪ ε)b*

Designing Regular Expressions

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- Let $L = \{\ w \in \Sigma^* \mid w$ contains at most one `a` $\}$.

`b*(a ∪ ε)b*`

Designing Regular Expressions

- Let $\Sigma = \{\texttt{a}, \texttt{b}\}$.
- Let $L = \{ w \in \Sigma^* \mid w$ contains at most one **a** $\}$.

b*(a ∪ ε)b*

bbbbabbb
bbbbbb
abbb
a

Designing Regular Expressions

- Let $\Sigma = \{\texttt{a}, \texttt{b}\}$.
- Let $L = \{ w \in \Sigma^* \mid w$ contains at most one `a` $\}$.

b*(a ∪ ε)b*

bbbbabbb
bbbbbb
abbb
a

Designing Regular Expressions

- Let $\Sigma = \{$`a`, `b`$\}$.
- Let $L = \{\ w \in \Sigma^* \mid w$ contains at most one `a` $\}$.

```
b*a?b*
```

```
bbbbabbb
bbbbbb
abbb
a
```

A More Elaborate Design

- Let Σ = { `a`, `.`, `@` }, where `a` represents "some letter."
- Let's make a regex for email addresses.

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cs103@cs.stanford.edu
first.middle.last@mail.site.org
dot.at@dot.com

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aa*(.aa*)*@
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aa*(.aa*)*@
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```
aa*(.aa*)*@aa*.aa*
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$$a^+ (.aa^*)^* @ aa^*.aa^* (.aa^*)^*$$

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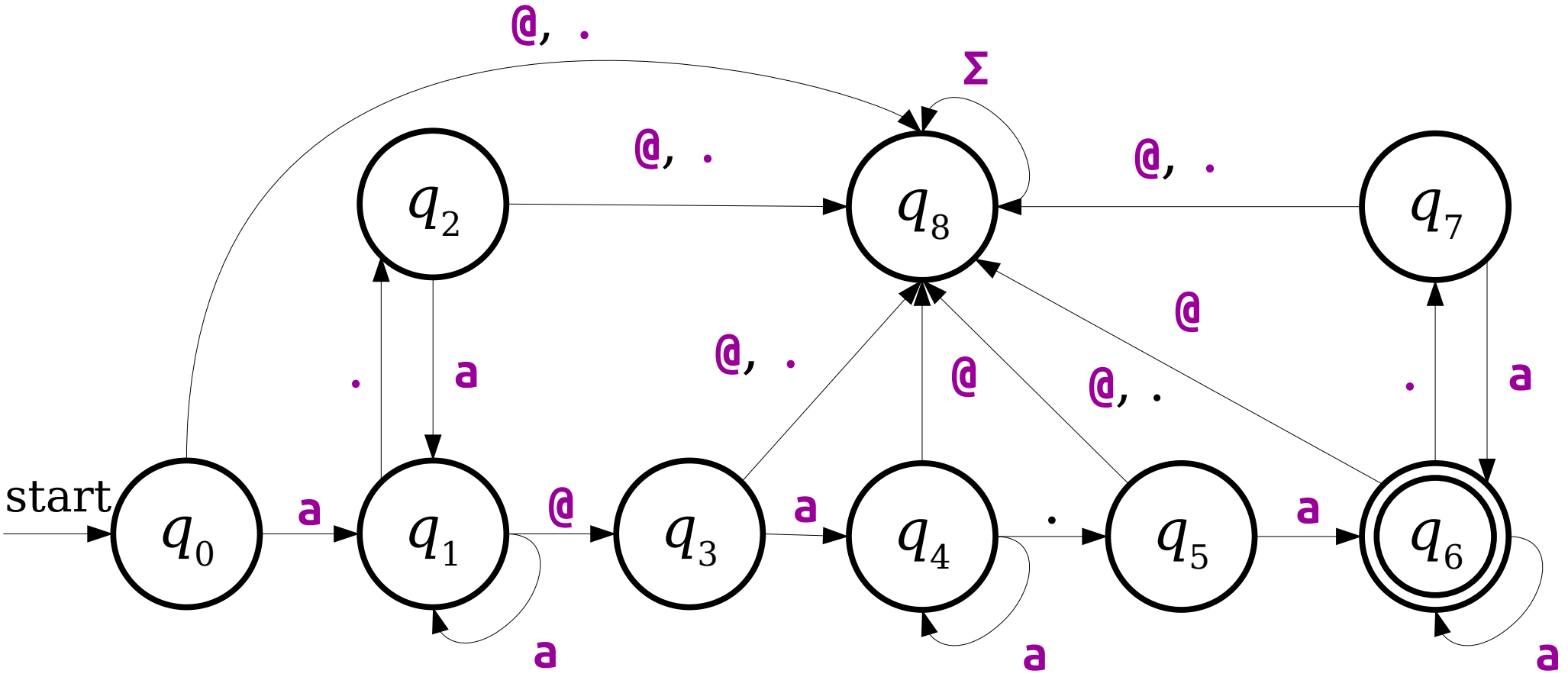
For Comparison

$$a^+(.a^+)^*@a^+(.a^+)^+$$



Shorthand Summary

- **`R`^{n}** is shorthand for **`RR`** … **`R`** (n times).
  - Edge case: define **`R`^{0}** = ε.
- **`Σ`** is shorthand for "any character in Σ."
- **`R?`** is shorthand for **`(R U ε)`**, meaning "zero or one copies of R."
- **`R`$^+$** is shorthand for **`RR*`**, meaning "one or more copies of R."

Let’s take a quick break!

Time-Out for Announcements!

Midterm Graded

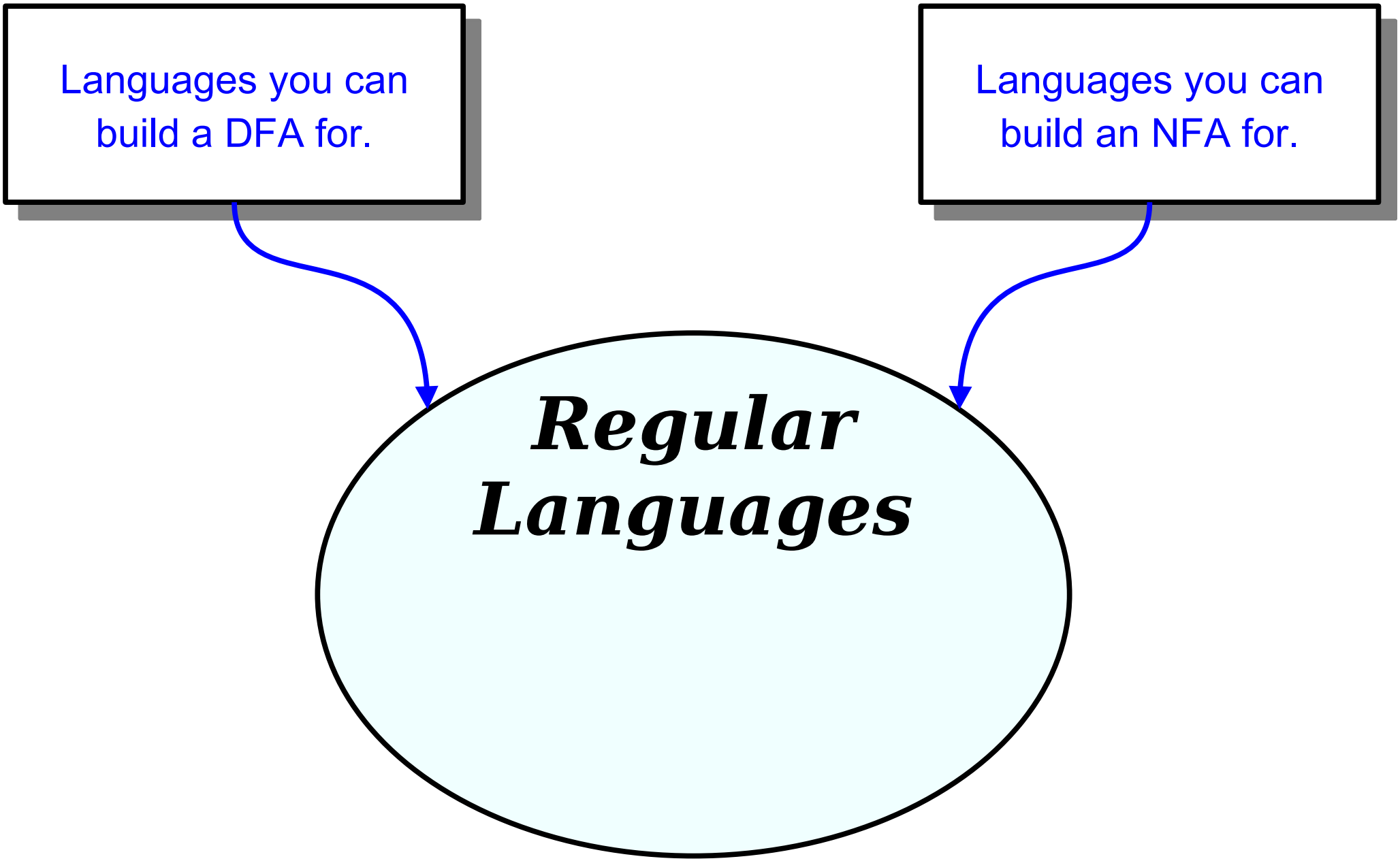
- We have finished grading the midterm exam.
  - Grades and feedback are now available on Gradescope.
  - Solution and grade distribution are now available on the course website.
- If you'd like to meet with us 1:1 to discuss the exam, or your general progress in the course, just reach out and we'll make it happen.
- Did we make a mistake? Regrades on Gradescope are open and are due in one week.

Problem Set Five

- Problem Set Four was due at 5:30PM on Sunday.
- Problem Set Five is currently out. It's due this Friday at 5:30PM.
  - Design DFAs and NFAs for a range of problems!
  - Explore formal language theory!
  - See some clever applications!

Back to CS103!

The Lay of the Land


Languages you can build a DFA for.
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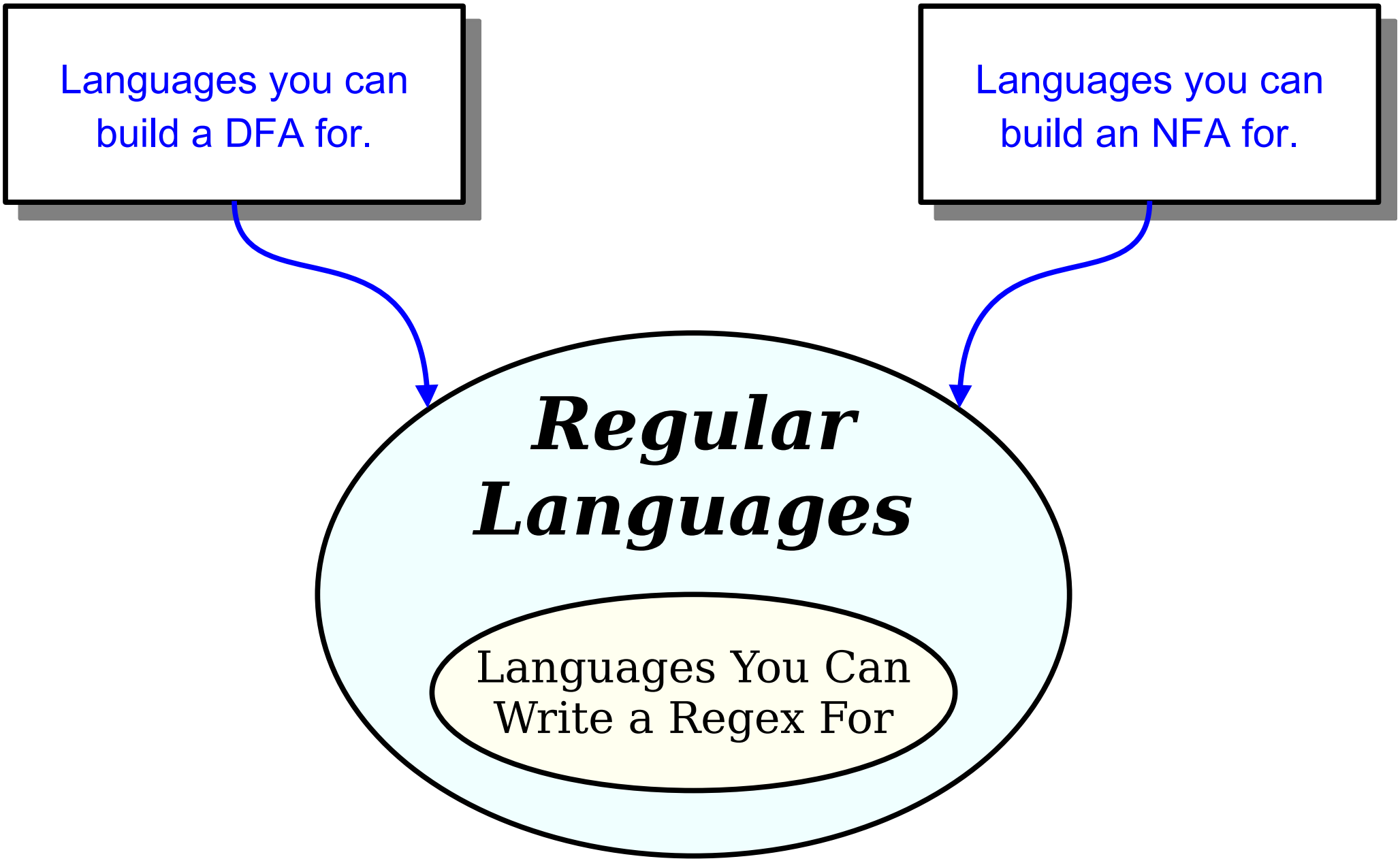
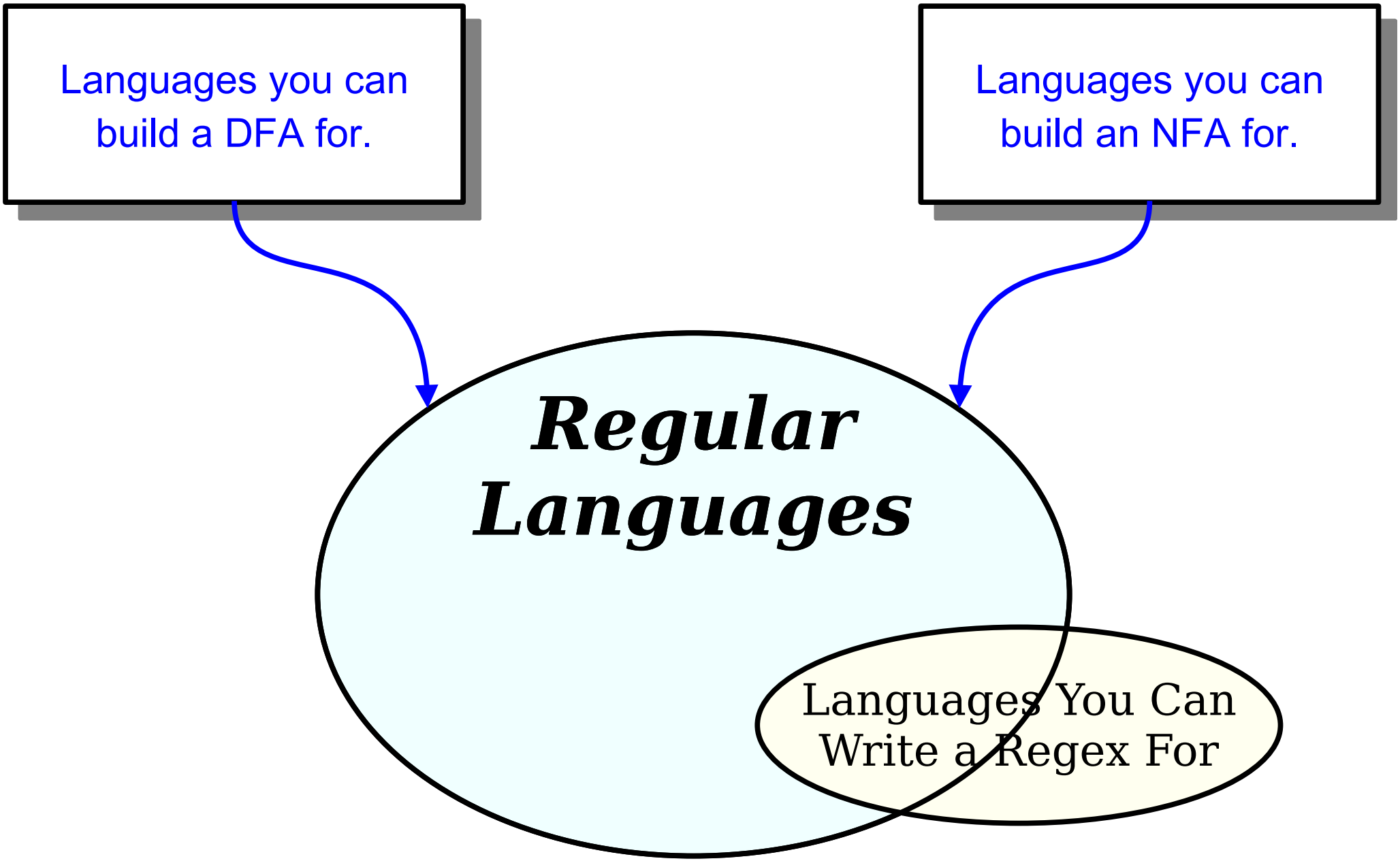
Regular Languages


Languages you can build a DFA for.
Languages you can build an NFA for.
Regular Languages
Languages You Can Write a Regex For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Languages you can build an NFA for.
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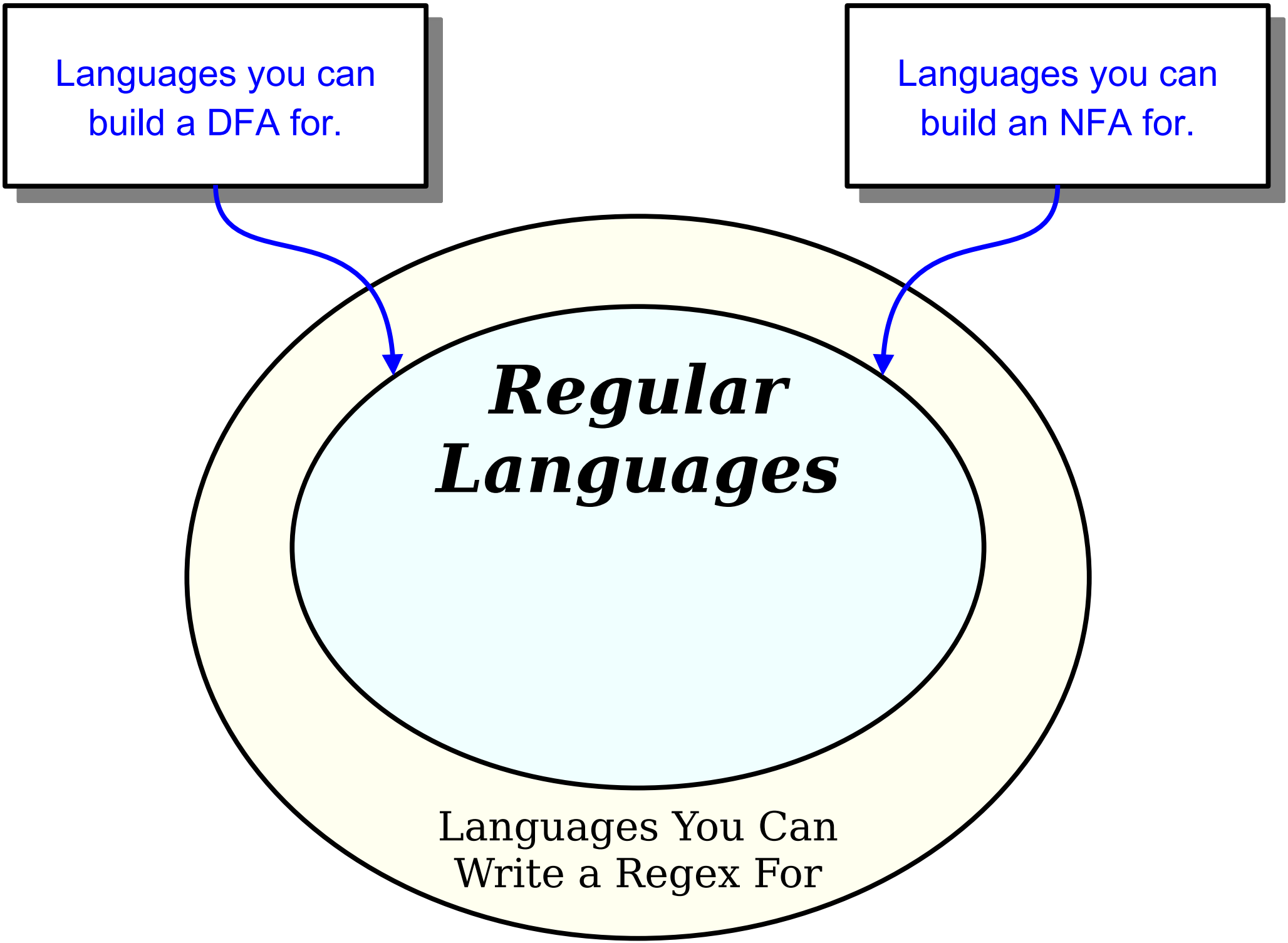
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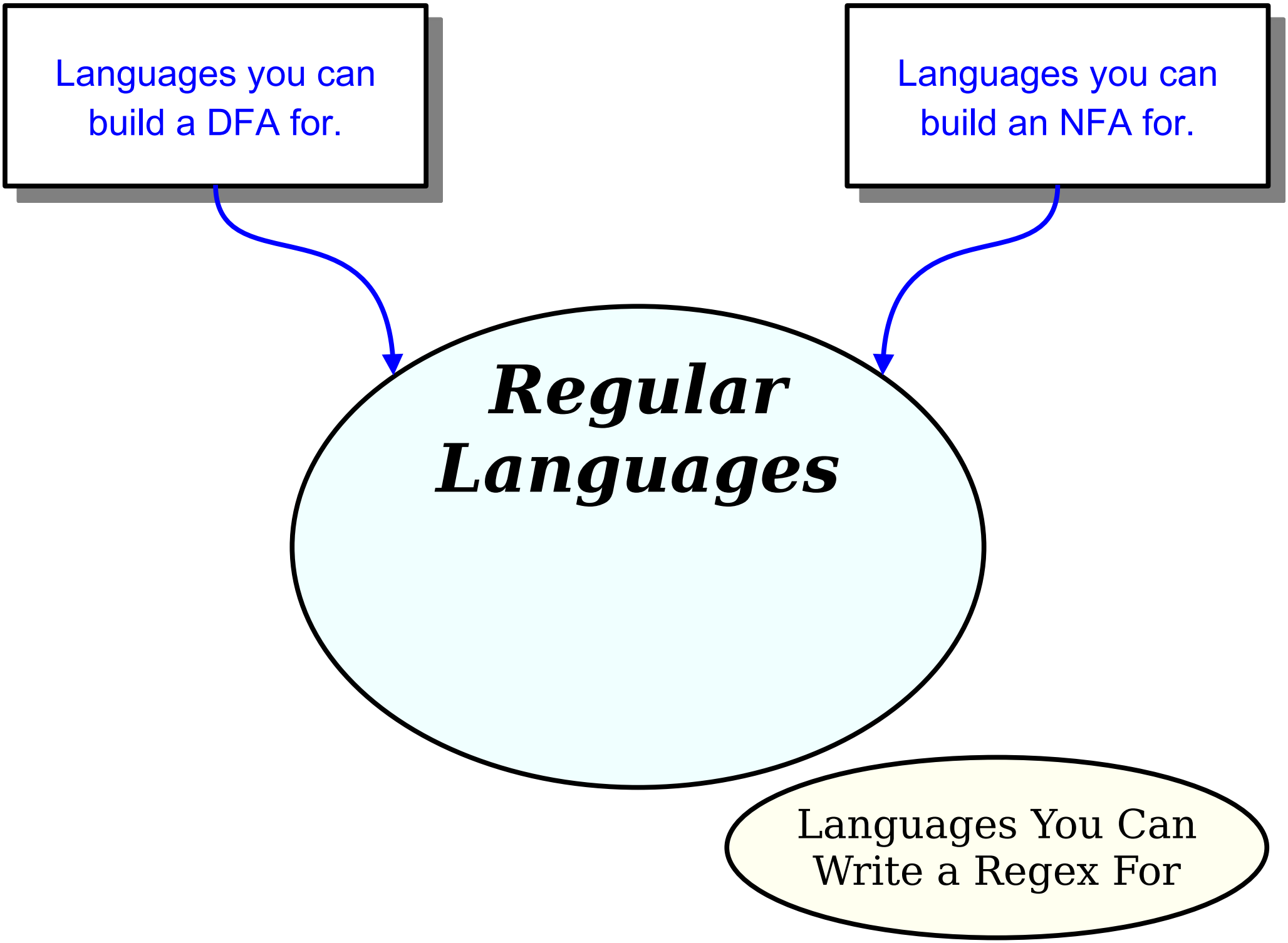
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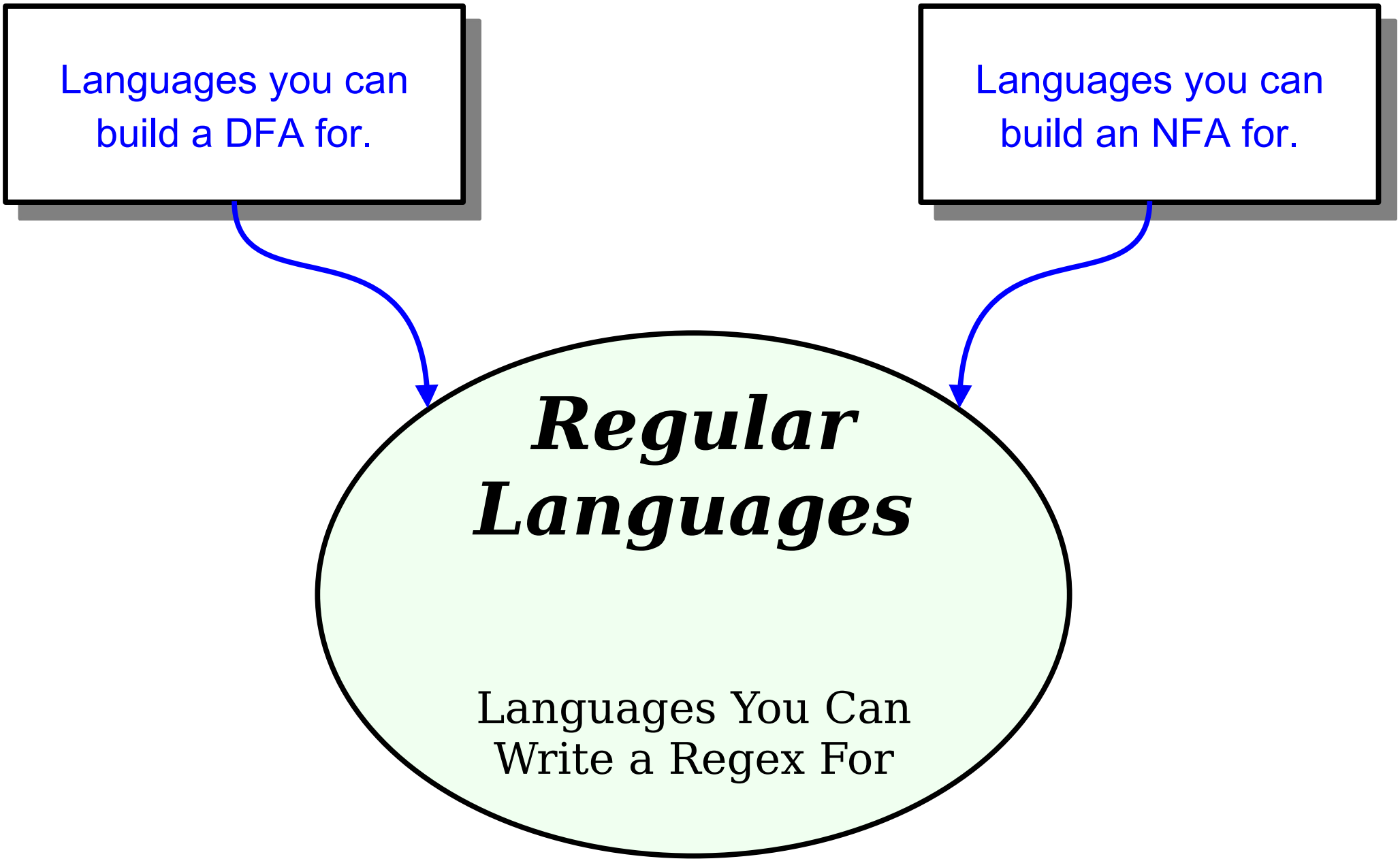
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The Power of Regular Expressions

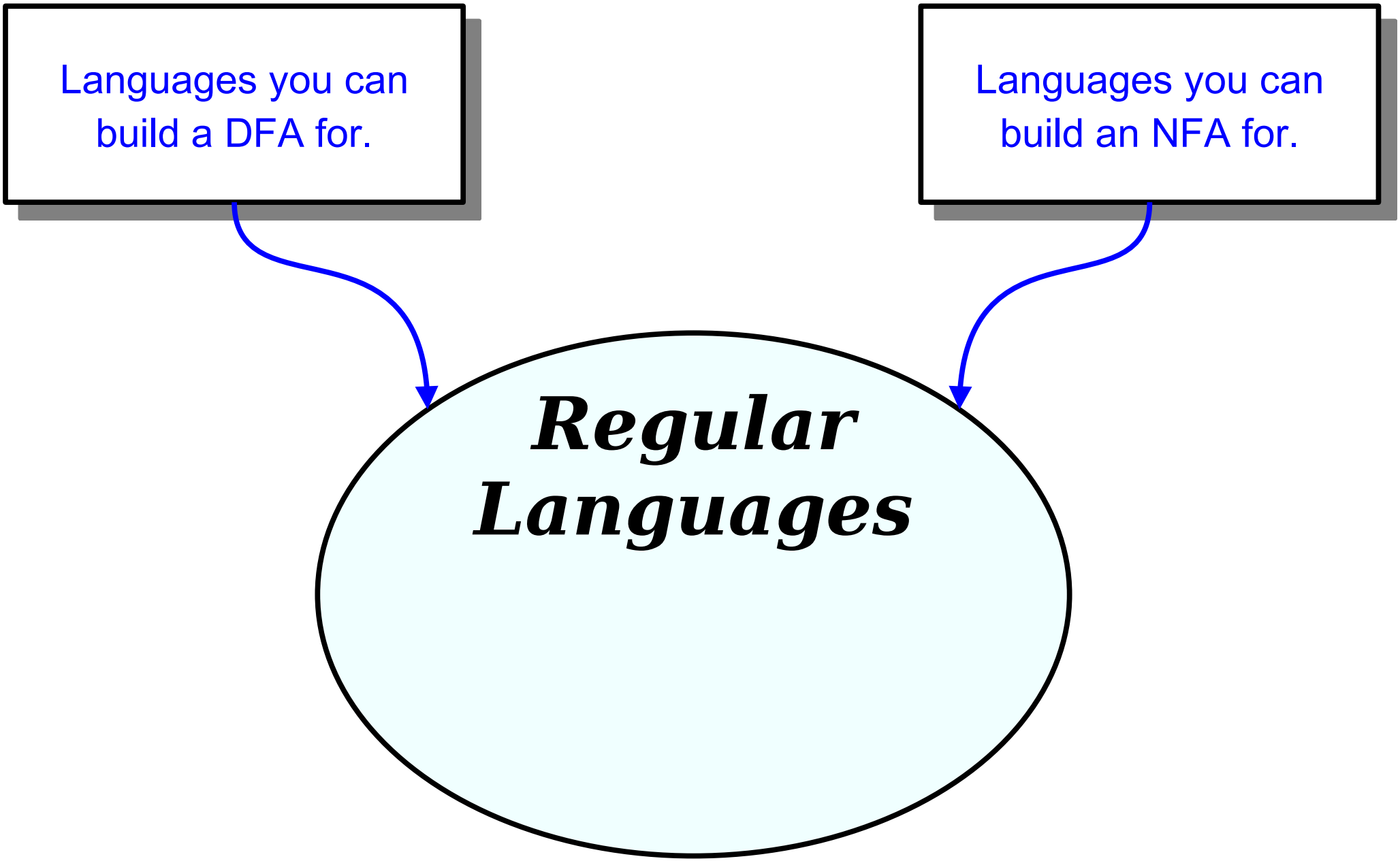
Theorem: If R is a regular expression, then $\mathscr{L}(R)$ is regular.

Proof idea: Use induction!

- The atomic regular expressions all represent regular languages.
- The combination steps represent closure properties.
- So anything you can make from them must be regular!

Thompson's Algorithm

- In practice, many regex matchers use an algorithm called ***Thompson's algorithm*** to convert regular expressions into NFAs (and, from there, to DFAs).
  - Read Sipser if you're curious!
- ***Fun fact:*** the "Thompson" here is Ken Thompson, one of the co-inventors of Unix!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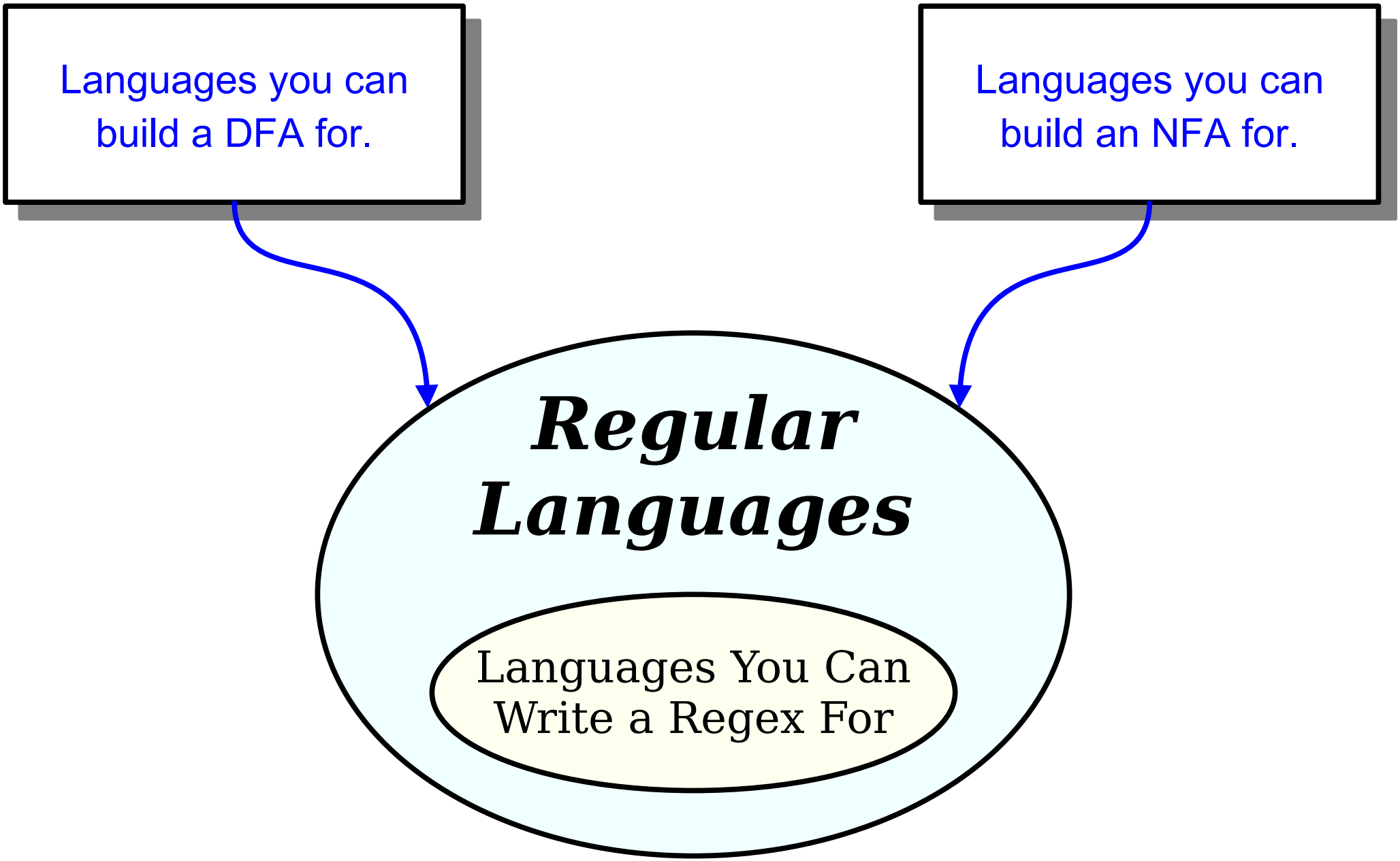
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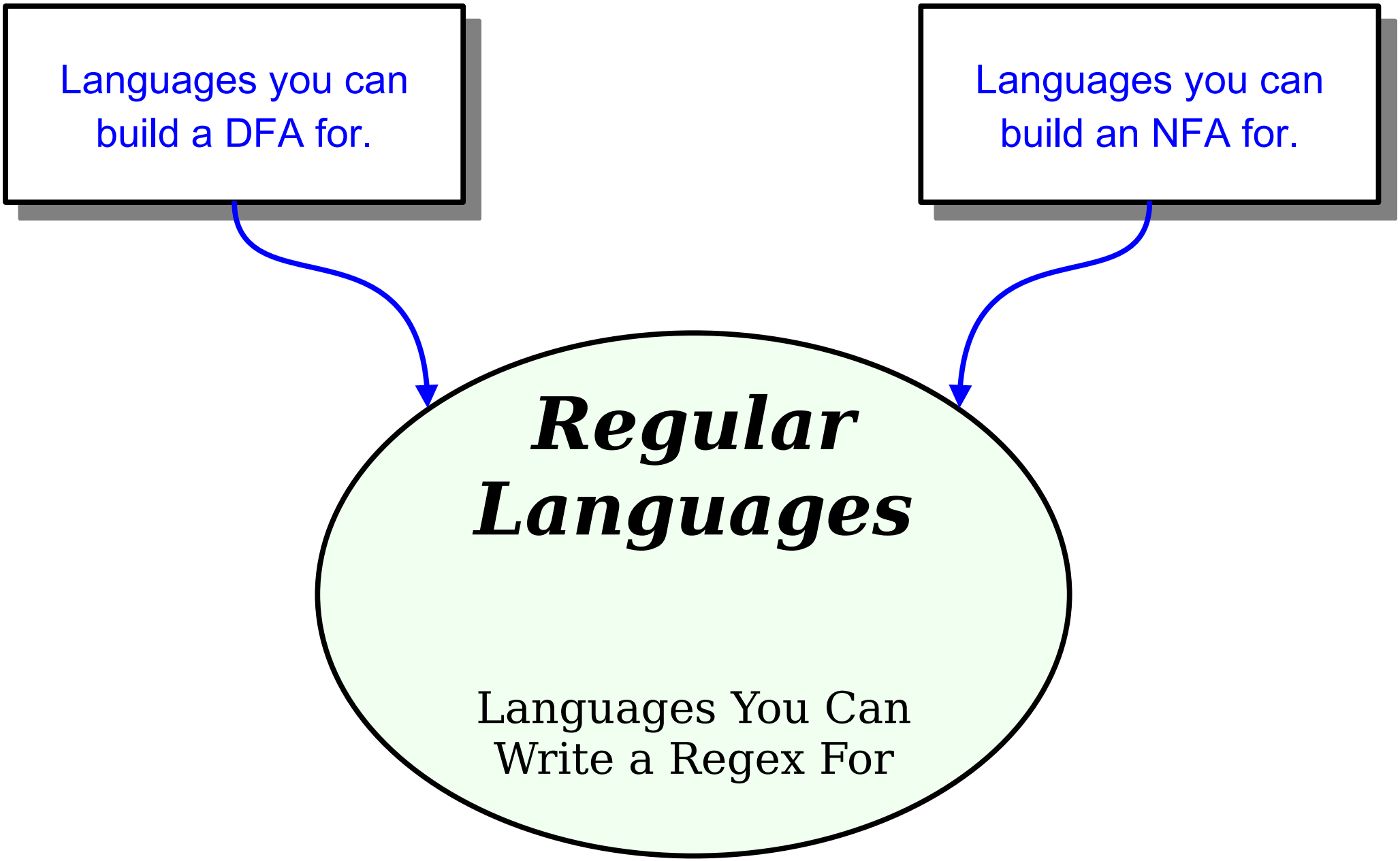
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The Power of Regular Expressions

Theorem: If L is a regular language, then there is a regular expression for L.

This is not obvious!

Proof idea: Show how to convert an arbitrary NFA into a regular expression.

Generalizing NFAs